Neutron-rich nuclei from the nuclear force

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ICNT workshop in MSU 2015/05/29

Nuclear force and Nuclear shell model

Shell model Hamiltonian

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k.$$

input

ε_i : single particle energies

$V_{ij,kl}$: two-body matrix elements

diagonalization →

output

Nuclear properties

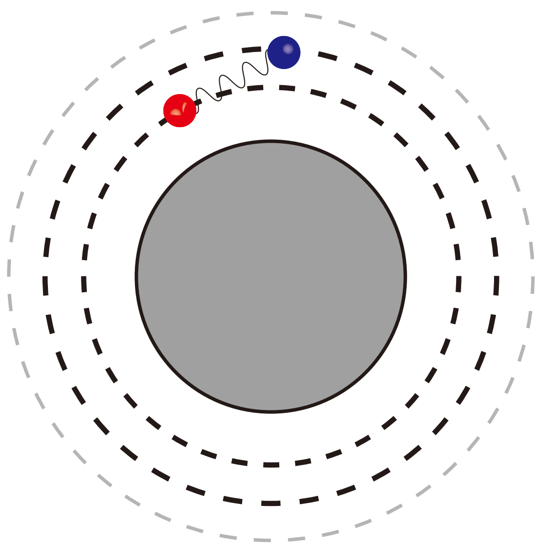
Binding energy, energy spectrum, transition probability , etc…

We would like to **derive** Shell model Hamiltonian based on Nuclear force and many-body theories

Nuclear force and Nuclear shell model

Shell model Hamiltonian $$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k.$$

Effective interaction in two-body space



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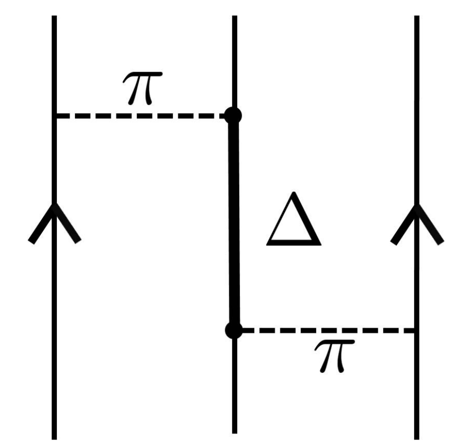
Three-body force



Reduce interaction to the model space perturbatively

Many-body perturbation theory

Fujita-Miyazawa interaction

Effective interaction and the model space

The effective interaction or the effective Hamiltonian have to satisfy the following properties

A. The interaction is designed for the selected subspace of the whole Hilbert space

B. The interaction yields the same physics as the original interaction (wave functions and eigenvalues)

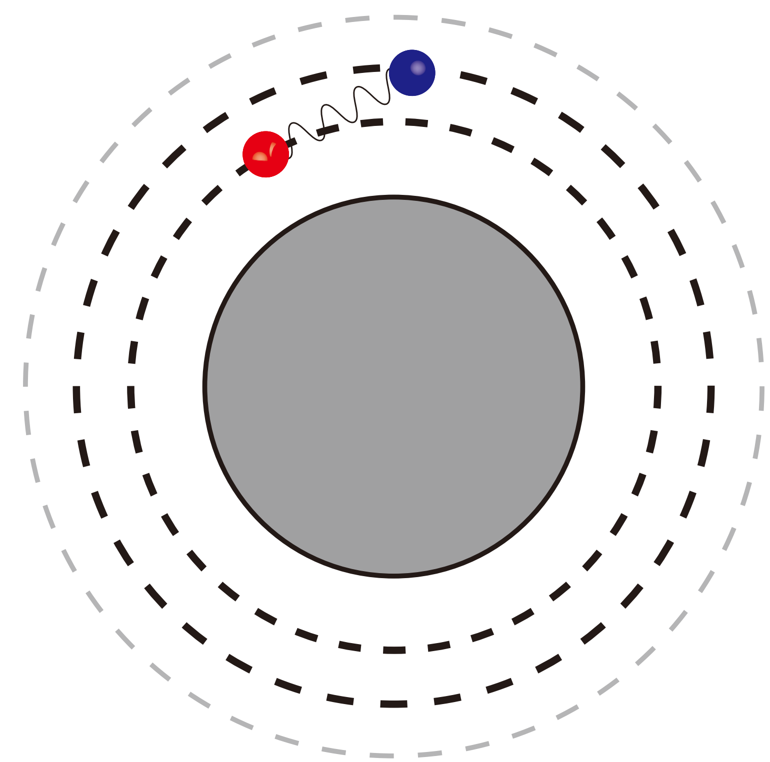
Hamiltonian with D-dimension

$$H = H_0 + V, \qquad H|\Psi_\lambda\rangle = E_\lambda|\Psi_\lambda\rangle, \quad \lambda = 1, \cdots, D.$$

Effective Hamiltonian with d-dimension (P-space)

$$H_{\text{eff}}|\phi_i\rangle = E_i|\phi_i\rangle, \quad i = 1, \cdots, d.$$

$$H_{\text{eff}} = \sum_{i=1}^{d} |\phi_i\rangle E_i \langle\tilde{\phi}_i|,$$



Notation: projection operator P and Q

P: projection to P-space

$$[P, H_0] = [Q, H_0] = 0.$$

$$P^2 = P, \quad Q^2 = Q$$

$$PQ = QP = 0,$$

$$[P, Q] = 0.$$

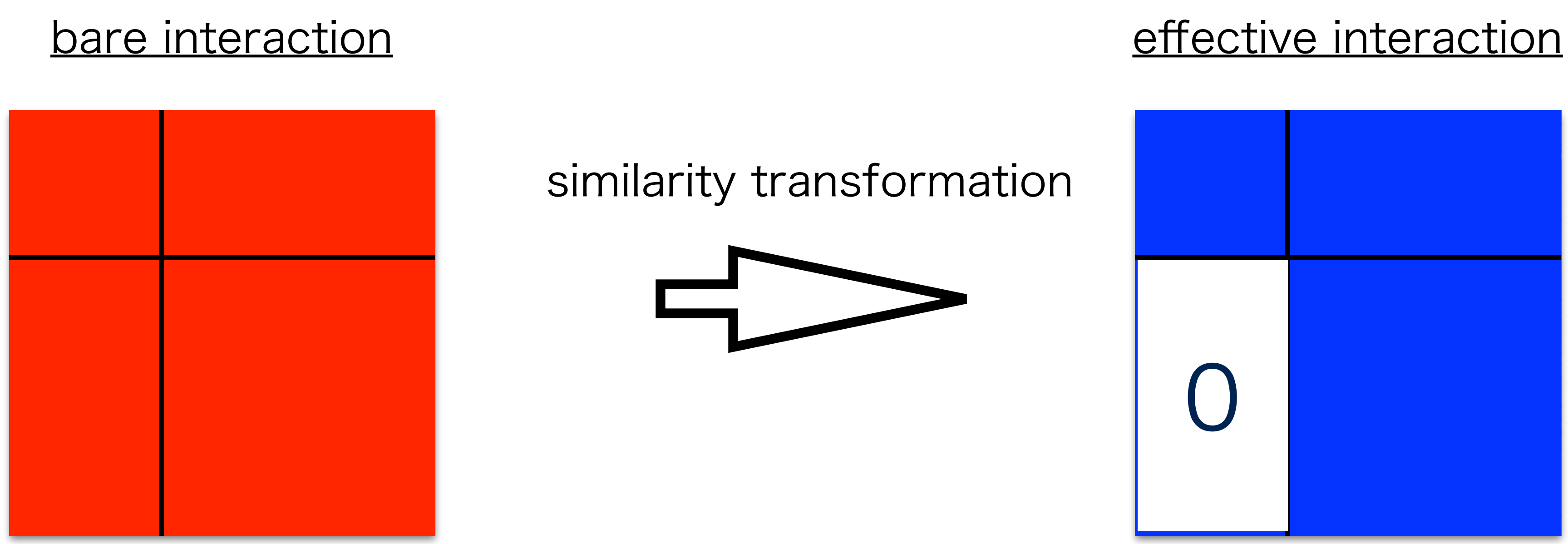
Decoupling equation for the KK method

similarity transformation to transform bare interaction to effective interaction

$$\mathcal{H} = e^{-\omega} H e^{\omega}, \quad Q\omega P = \omega.$$

decoupling condition

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$



$$H_{\text{eff}} = P\mathcal{H}P$$

$$V_{\text{eff}} = PVP + PVQ\omega.$$

It is needed to solve non-linear decoupling equation

Formal solution of decoupling equation (KK method)

Assumption: the model space is degenerate

$$PH_0P = \epsilon_0 P.$$

A possible solution of decoupling equation

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$

$$(\epsilon_0 - QHQ)\omega = QVP - \omega PVP - \omega PVQ\omega.$$

$$\begin{aligned}\omega &= \frac{1}{\epsilon_0 - QHQ}\left(QVP - \omega\left(PVP + PVQ\omega\right)\right) \\ &= \frac{1}{\epsilon_0 - QHQ}\left(QVP - \omega V_{\mathrm{eff}}\right),\end{aligned}$$

Introduce Q-box defined as an operator in P-space

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP,$$

$$\hat{Q}_k(E) = \frac{1}{k!}\frac{\mathrm{d}^k\hat{Q}(E)}{\mathrm{d}E^k}.$$

$\longrightarrow$ $$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty}\hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

Iterative equation for deriving the Effective interaction for degenerate model space

Derivation via the time-dependent perturbation theory

Time-dependent operator in interaction picture

$$U(t,t') = \lim_{\epsilon\to 0} \lim_{t'\to -\infty(1-i\epsilon)} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t'}^{t} \mathrm{d}t_1 \int_{t'}^{t} \mathrm{d}t_2 \cdots \int_{t'}^{t} \mathrm{d}t_n T[H_1(t_1)H_1(t_2)\cdots H_1(t_n)].$$

Parent state: projection of P-space eigen-function ψ_α to P-space

$$|\rho_\lambda\rangle = \sum_{\alpha=1}^{d} C_\alpha^{(\lambda)} |\psi_\alpha\rangle. \qquad \langle\rho_\lambda|P\Psi_\mu\rangle = 0 \qquad (\lambda \neq \mu = 1, 2, \cdots, D).$$

$$\frac{|\Psi_\lambda\rangle}{\langle\rho_\lambda|\Psi_\lambda\rangle} = \lim_{\epsilon\to 0} \lim_{t'\to -\infty(1-i\epsilon)} \frac{U(0,t')|\rho_\lambda\rangle}{\langle\rho_\lambda|U(0,t')|\rho_\lambda\rangle} \quad \longrightarrow \quad H\frac{U(0,-\infty)|\rho_\lambda\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle} = E_\lambda \frac{U(0,-\infty)|\rho_\lambda\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}.$$

$$\sum_{\alpha=1}^{D} C_\alpha^{(\lambda)} H \frac{U(0,-\infty)|\psi_\alpha\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle} = \sum_{\beta=1}^{D} C_\beta^{(\lambda)} E_\lambda \frac{U(0,-\infty)|\psi_\beta\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}.$$

$HU(0,-\infty)$ is nearly equal to effective interaction Heff

Effective interaction Veff include Q-box and its infinite order repetition

$$V_{\mathrm{eff}} = \hat{Q}(\epsilon_0) - \hat{Q}'(\epsilon_0)\int \hat{Q}(\epsilon_0) + \hat{Q}'(\epsilon_0)\int \hat{Q}(\epsilon_0)\int \hat{Q}(\epsilon_0)\cdots$$

$$\begin{aligned}\hat{Q}(E) &= PVP + PVQ\frac{1}{E-QHQ}QVP \\ &= PVP + PVQ\frac{1}{E-QH_0Q}QVP + PVQ\frac{1}{E-QH_0Q}QVQ\frac{1}{E-QH_0Q}QVP + \cdots\end{aligned}$$

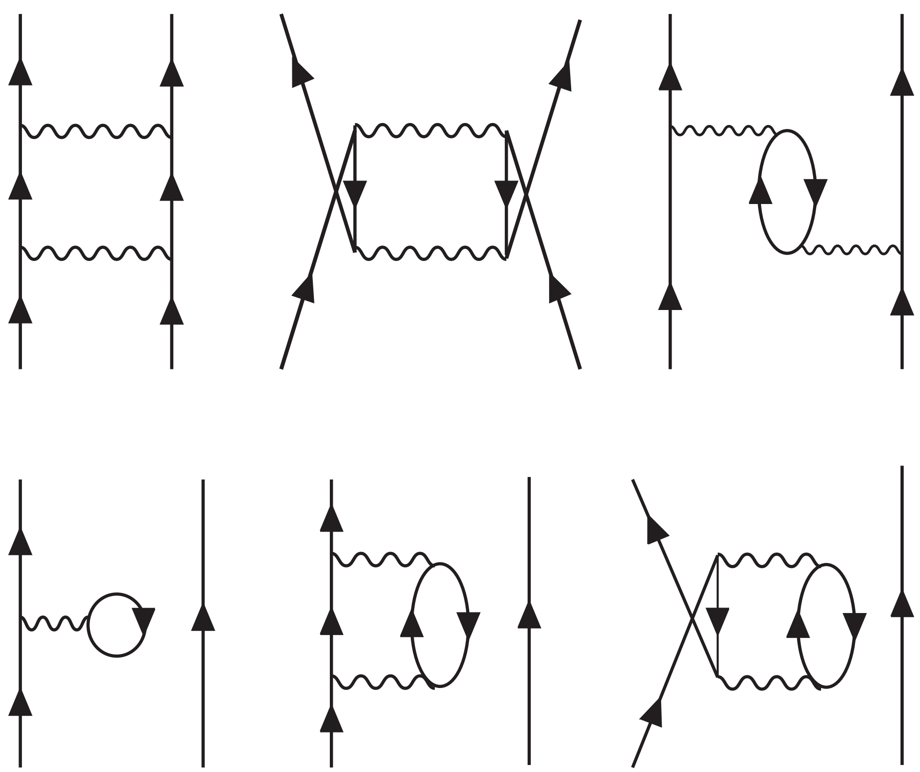
Q-box expansion

Q-box is the ingredient of effective interaction and approximated by perturbation theory

$$\boxed{\begin{aligned}\hat{Q}(E) &= PVP + PVQ\frac{1}{E - QHQ}QVP \\ &= PVP + PVQ\frac{1}{E - QH_0Q}QVP + PVQ\frac{1}{E - QH_0Q}QVQ\frac{1}{E - QH_0Q}QVP + \cdots\end{aligned}}$$

P is proj. operator to model space

Q=1-P



Diagrams appearing in 2nd order Q-box

Folded diagram technique (Kuo-Krenciglowa method) to include the infinite time repetitions of Q-box (but only for the degenerate model space)

$$V_{\rm eff} = \hat{Q}(\epsilon_0) - \hat{Q}'(\epsilon_0)\int\hat{Q}(\epsilon_0) + \hat{Q}'(\epsilon_0)\int\hat{Q}(\epsilon_0)\int\hat{Q}(\epsilon_0)\cdots,$$

$$V_{\rm eff}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty}\hat{Q}_k(\epsilon_0)\{V_{\rm eff}^{(n-1)}\}^k.$$

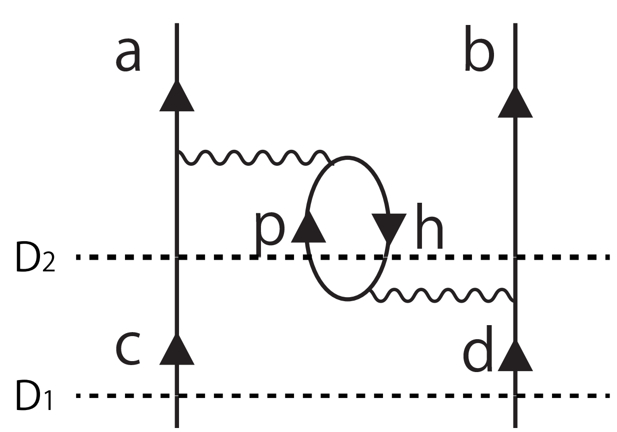
$$\hat{Q}_k(E) = \frac{1}{k!}\frac{{\rm d}^k\hat{Q}(E)}{{\rm d}E^k}.$$

Divergent problem of Q-box in non-degenerate model space

(A) Folded diagram theory requires assumption that the model space is **degenerate**

(B) Naive perturbation theory leads a **divergence** in non-degenerate model space

Example



$$\frac{V_{ah,cp}V_{pb,hd}}{(\epsilon_c+\epsilon_d)-\epsilon_c-\epsilon_p+\epsilon_h-\epsilon_b}$$

→ **Energy denominator is zero when $\varepsilon_d - \varepsilon_b = \varepsilon_p - \varepsilon_h$**

We need a theory which satisfies

(a) The assumption of degenerate model space is **removed**

(b) **Avoid** the divergence appearing in Q-box diagrams

→ **EKK method as a re-summation scheme of KK method**

Decoupling equation for the EKK method (formal solution)

Decoupling equation

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$

Introduce energy parameter E

$$(E - QHQ)\omega = QVP - \omega P\tilde{H}P - \omega PVQ\omega,$$
$$\tilde{H} = H - E$$

$$\longrightarrow \quad \tilde{H}_{\mathrm{eff}}^{(n)} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\mathrm{eff}}^{(n-1)}\}^k, \qquad H_{\mathrm{BH}}(E) = PHP + PVQ\frac{1}{E - QHQ}QVP.$$

$$\tilde{H}_{\mathrm{eff}} = H_{\mathrm{eff}} - E, \quad \tilde{H}_{\mathrm{BH}}(E) = H_{\mathrm{BH}}(E) - E,$$

Points:

1. Arbitrary energy parameter E is introduced

   → results do not depend on the choice of E
2. Veff is substituted by Heff
3. Q-box and its derivatives are not changed, but evaluated at E

Extended KK method as a re-summation of the perturbative series

EKK method is derived with the following re-interpretation of the Hamiltonian

$$
\begin{aligned}
H &= H_0' + V' \\
&= \begin{pmatrix} E & 0 \\ 0 & QH_0Q \end{pmatrix} + \begin{pmatrix} P\tilde{H}P & PVQ \\ QVP & QVQ \end{pmatrix},
\end{aligned}
$$

New parameter E (arbitrary parameter)

Change PH_0P part of the unperturbed Hamiltonian

KK method

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP$$

$$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

EKK method

$$H_{\mathrm{BH}}(E) = PHP + PVQ\frac{1}{E - QHQ}QVP.$$

$$\tilde{H}_{\mathrm{eff}}^{(n)} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\mathrm{eff}}^{(n-1)}\}^k.$$

- One can take E so as to avoid the divergence !
- Final result does not depends on E.

Extended KK method as an analogy of Taylor series

KK method

$$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

$$e^x = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} x^k$$

Taylor expansion
around x=0

EKK method

$$\tilde{H}_{\mathrm{eff}}^{(n)} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\mathrm{eff}}^{(n-1)}\}^k.$$

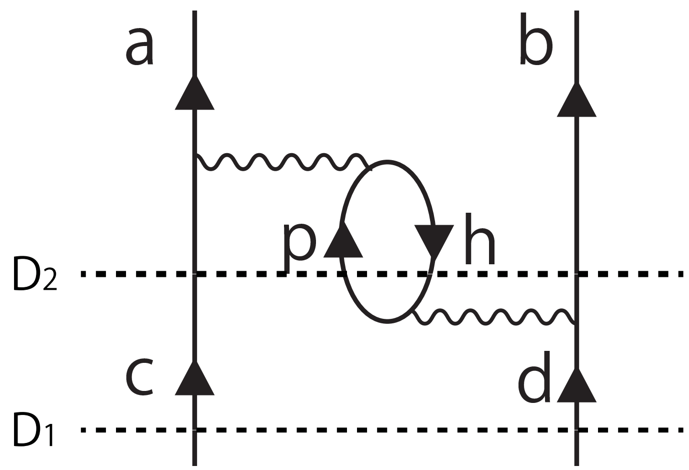
$$e^x = e^E + \sum_{k=1}^{\infty} \frac{e^E}{k!} (x - E)^k$$

Taylor expansion
around x=E

→ Result does not depend on E

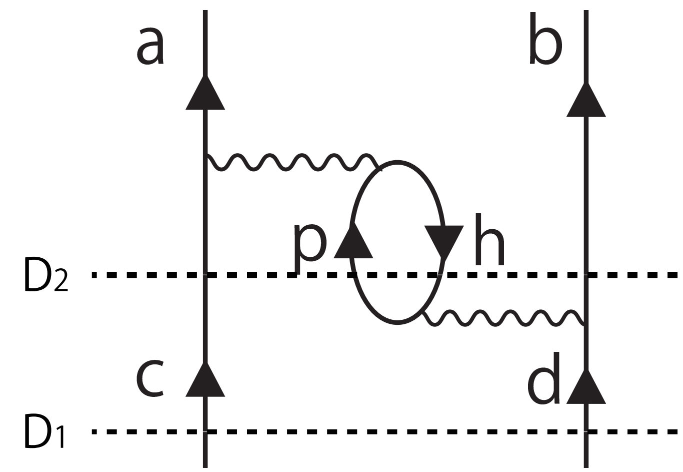
Example: EKK method avoids the divergences

EKK method



$$\frac{V_{ah,cp}V_{pb,hd}}{E - \epsilon_c - \epsilon_b - \epsilon_p + \epsilon_h}$$

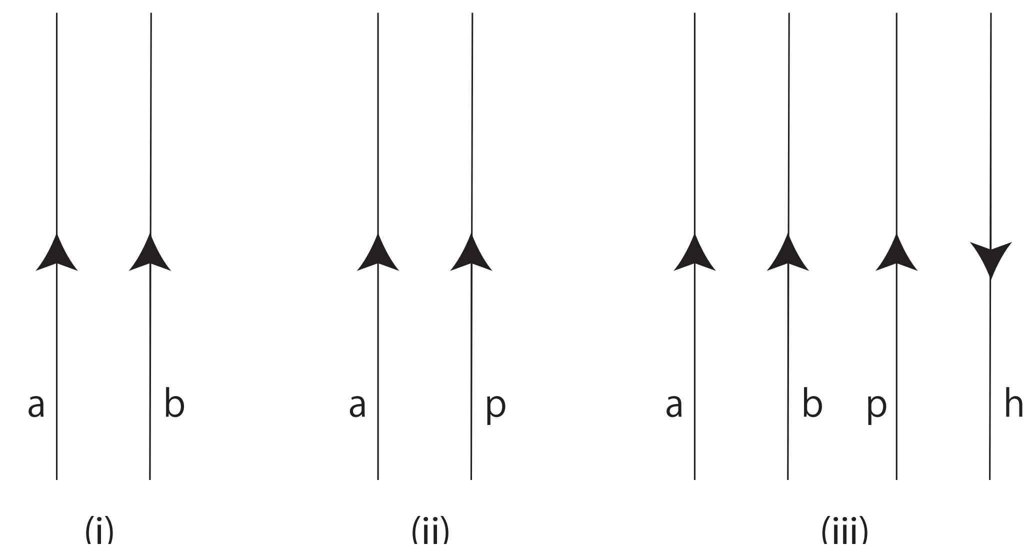
KK method



$$\frac{V_{ah,cp}V_{pb,hd}}{(\epsilon_c + \epsilon_d) - \epsilon_c - \epsilon_p + \epsilon_h - \epsilon_b}$$

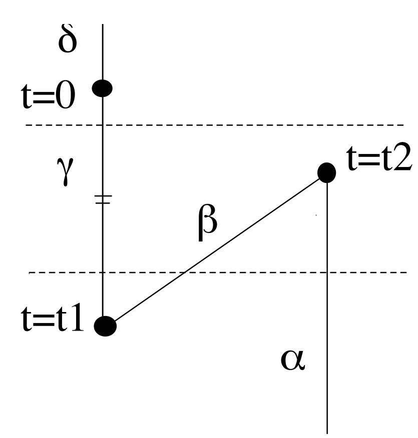
- We can choose E to avoid divergence !
- Note that the choice of E is arbitrary and should give the same result if the Q-box is calculated without any approximation.
- Inversely, E-dependence is a **measure of error** coming from the approximation

Diagrams appearing in EKK method



(i)	$\lvert\psi_i(t)\rangle$	$= e^{-iH_0' t}\lvert\psi_i\rangle$	$= e^{-iEt}\lvert\psi_i\rangle$	P-space
(ii)	$\{a_a^\dagger a_p^\dagger \lvert c\rangle\}(t)$	$= e^{-iH_0' t}\{a_a^\dagger a_p^\dagger \lvert c\rangle\}$	$= e^{-i(\epsilon_a+\epsilon_p)t} a_a^\dagger a_p^\dagger \lvert c\rangle,$	Q-space
(iii)	$\{a_a^\dagger a_b^\dagger a_p^\dagger a_h \lvert c\rangle\}(t)$	$= e^{-iH_0' t}\{a_a^\dagger a_b^\dagger a_p^\dagger a_h \lvert c\rangle\}$	$= e^{-i(\epsilon_a+\epsilon_b+\epsilon_p-\epsilon_h)t} a_a^\dagger a_b^\dagger a_p^\dagger a_h \lvert c\rangle,$	Q-space

The argument of folded diagram is the same
→ derivatives indicate the folded diagram contribution



$$= \frac{V_{\alpha\beta} V_{\beta\gamma} V_{\gamma\delta}}{(\epsilon_\alpha - \epsilon_\gamma - (\epsilon_\alpha - \epsilon_\beta))(\epsilon_\alpha - \epsilon_\gamma)}$$

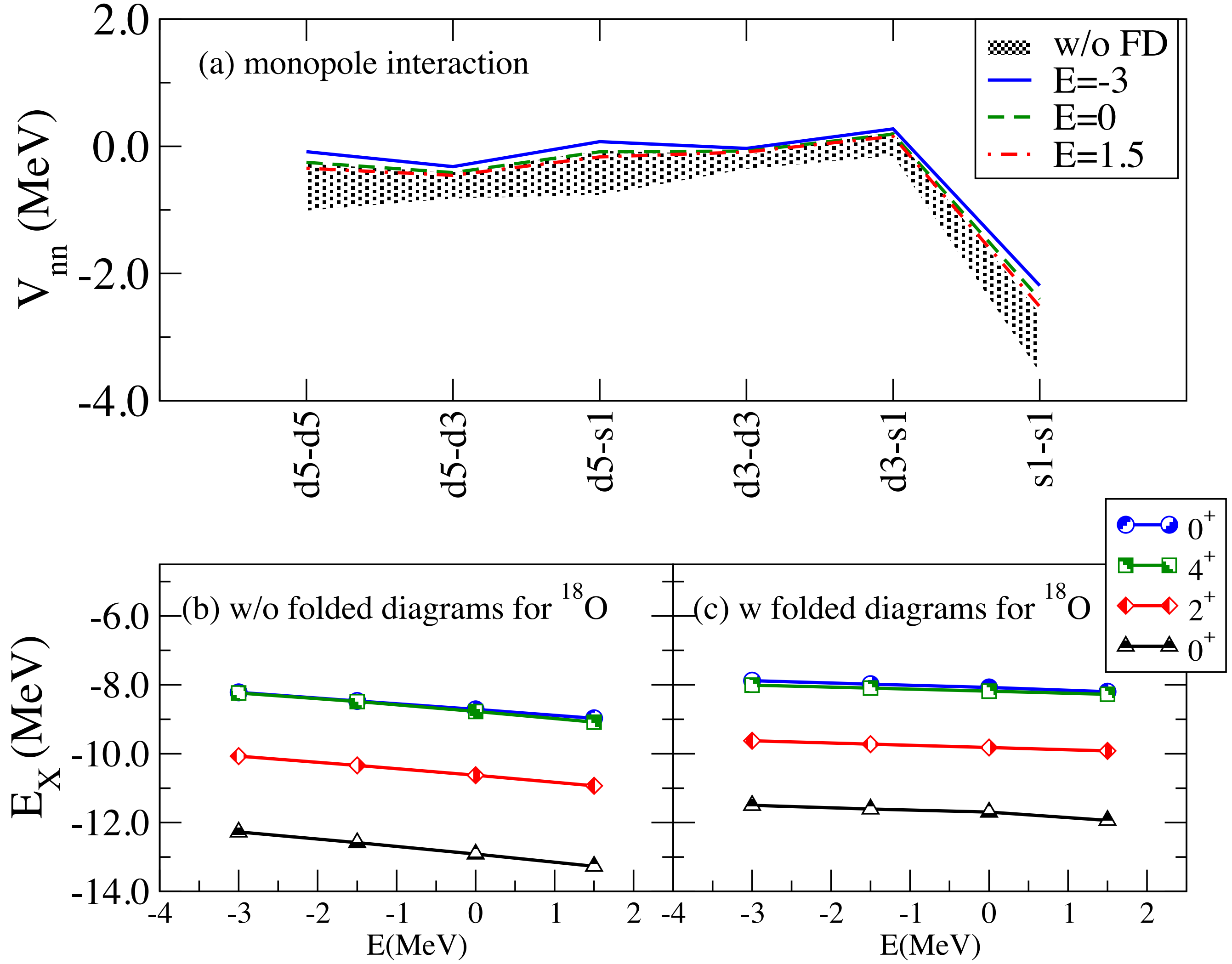
$$= V_{\alpha\beta} V_{\beta\gamma} V_{\gamma\delta} \frac{\Big((\epsilon_\alpha - \epsilon_\gamma) - (\epsilon_\alpha - \epsilon_\beta)\Big)^{-1} - (\epsilon_\alpha - \epsilon_\gamma)^{-1}}{\epsilon_\alpha - \epsilon_\beta}$$

in the limit of $\epsilon_\beta \to \epsilon_\alpha$

$$= \frac{\mathrm{d}}{\mathrm{d}\omega} \left(\frac{V_{\beta\gamma} V_{\gamma\delta}}{\omega - \epsilon_\gamma} \right)_{\omega=\alpha} \times V_{\alpha\beta}$$

Effective interaction in degenerate sd-shell

Neutron-Neutron channel



Monopole part of the interaction between the orbit j and j'

$$V_{\mathrm{eff}}{}^{T}_{j,j'} = \frac{\sum_J (2J+1)\langle jj'|V_{\mathrm{eff}}|jj'\rangle_{JT}}{\sum_J (2J+1)}.$$

Energy levels with respect to ^{16}O

Single particle energies are taken from phenomenological interaction USD

- w/o Folded diagram contribution, the monopole and the energy levels are depend on E, but the dependence is disappear when the folded diagram contribution added
- Agrees with the theoretical consideration that the results does not depend on E

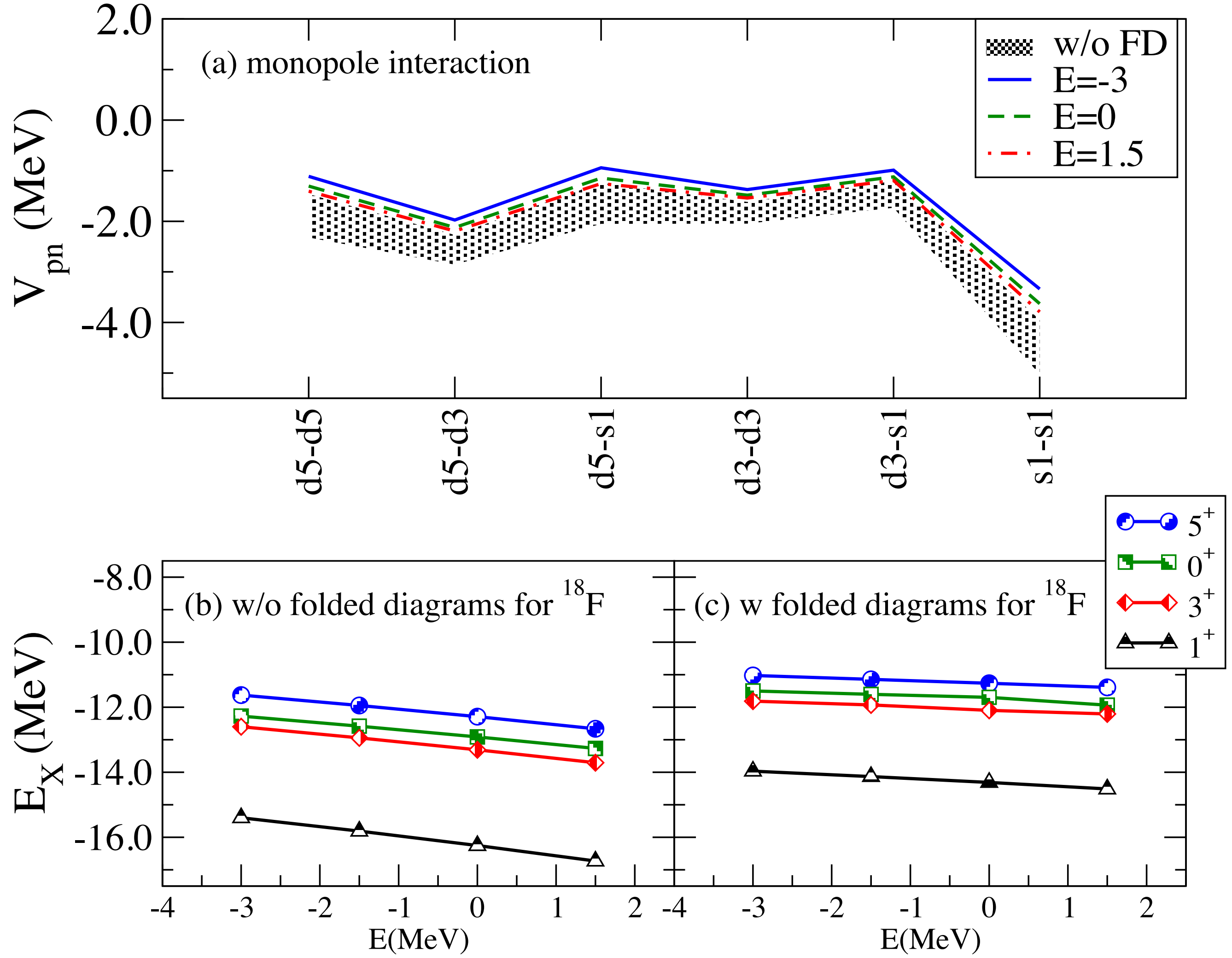
reminder

$$\tilde{H}^{(n)}_{\mathrm{eff}} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}^{(n-1)}_{\mathrm{eff}}\}^k,$$

$$\tilde{H}_{\mathrm{eff}} = H_{\mathrm{eff}} - E, \quad \tilde{H}_{\mathrm{BH}}(E) = H_{\mathrm{BH}}(E) - E,$$

Effective interaction in degenerate sd-shell

Proton-Neutron channel



Monopole part of the interaction between the orbit j and j'

$$V_{\text{eff}}{}^{T}_{j,j'} = \frac{\sum_J (2J+1)\langle jj'|V_{\text{eff}}|jj'\rangle_{JT}}{\sum_J (2J+1)}.$$

Energy levels with respect to ^{16}O

Single particle energies are taken from phenomenological interaction USD

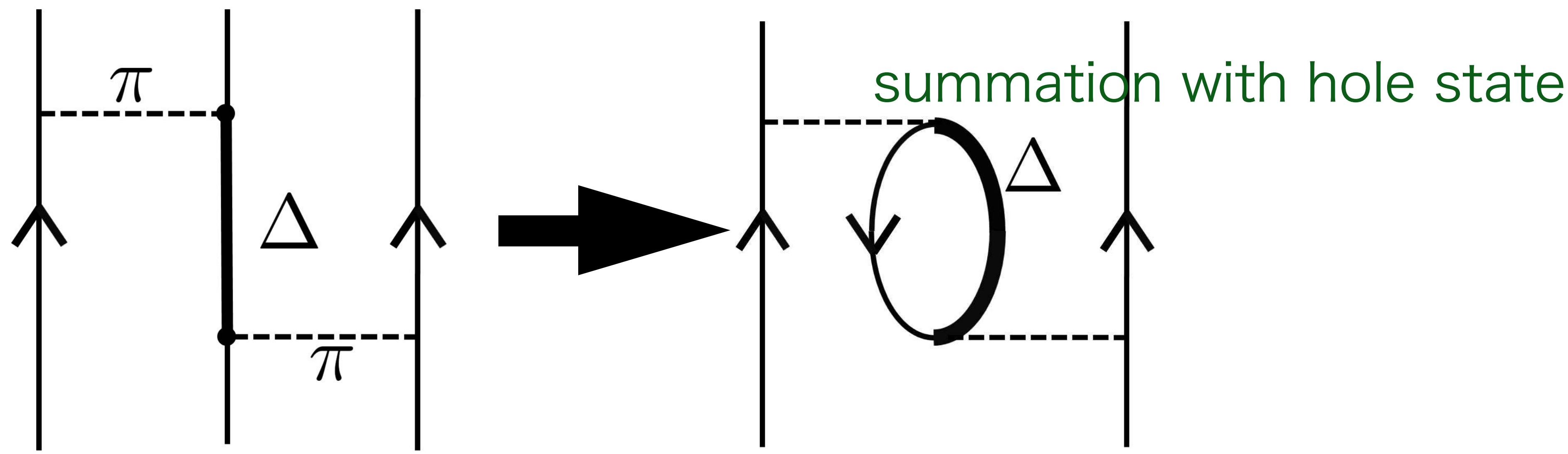
- The same observation as NN channel
- 1^+ state is slightly more dependent on E than other states, but folded diagram contribution reduce the E-dependence by around 80 to 90 percent

reminder

$$\tilde{H}^{(n)}_{\text{eff}} = \tilde{H}_{\text{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}^{(n-1)}_{\text{eff}}\}^k,$$

$$\tilde{H}_{\text{eff}} = H_{\text{eff}} - E, \quad \tilde{H}_{\text{BH}}(E) = H_{\text{BH}}(E) - E,$$

3N interaction



Fujita-Miyazawa type
3N interaction

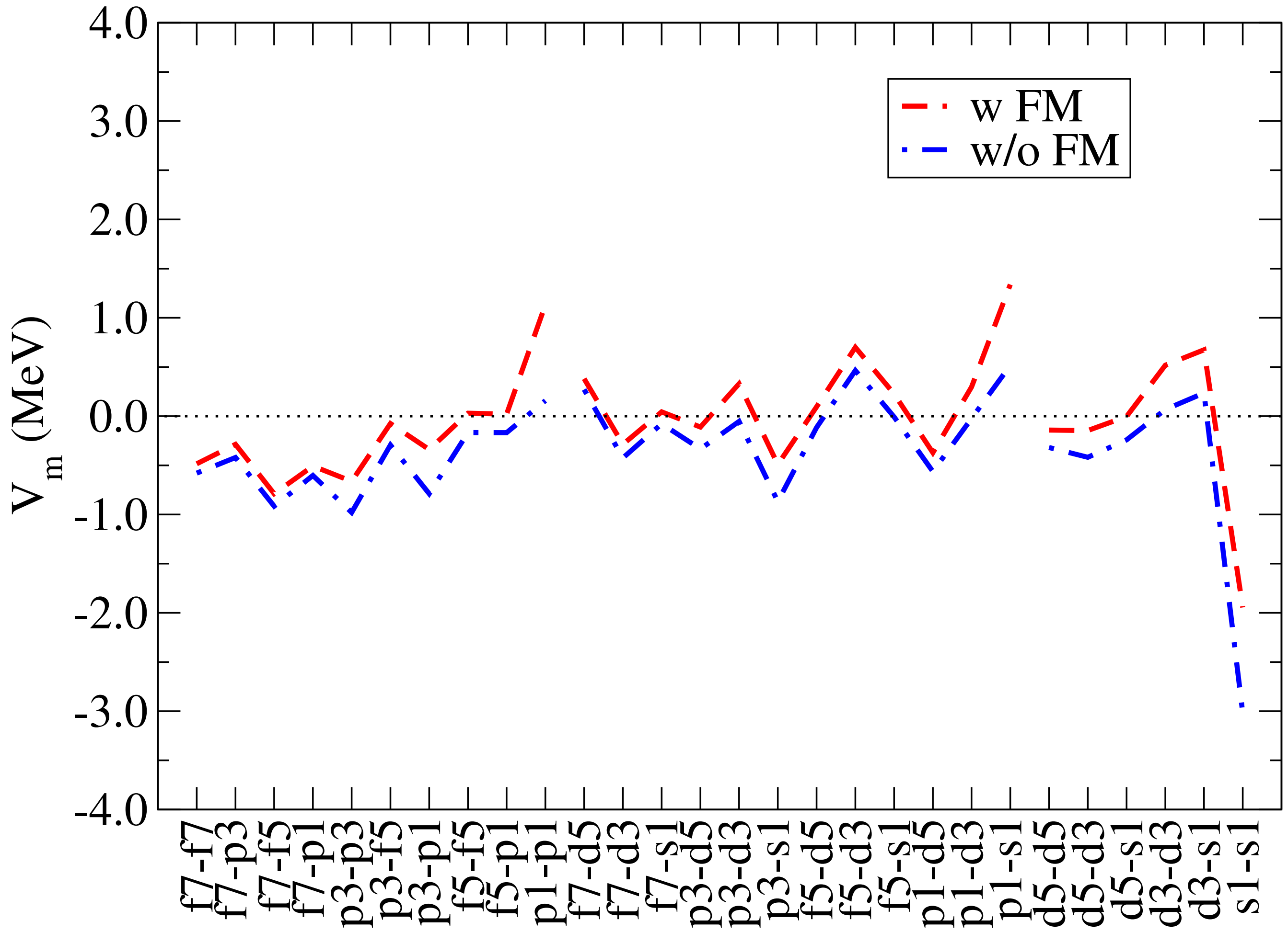
Effective
2N interaction

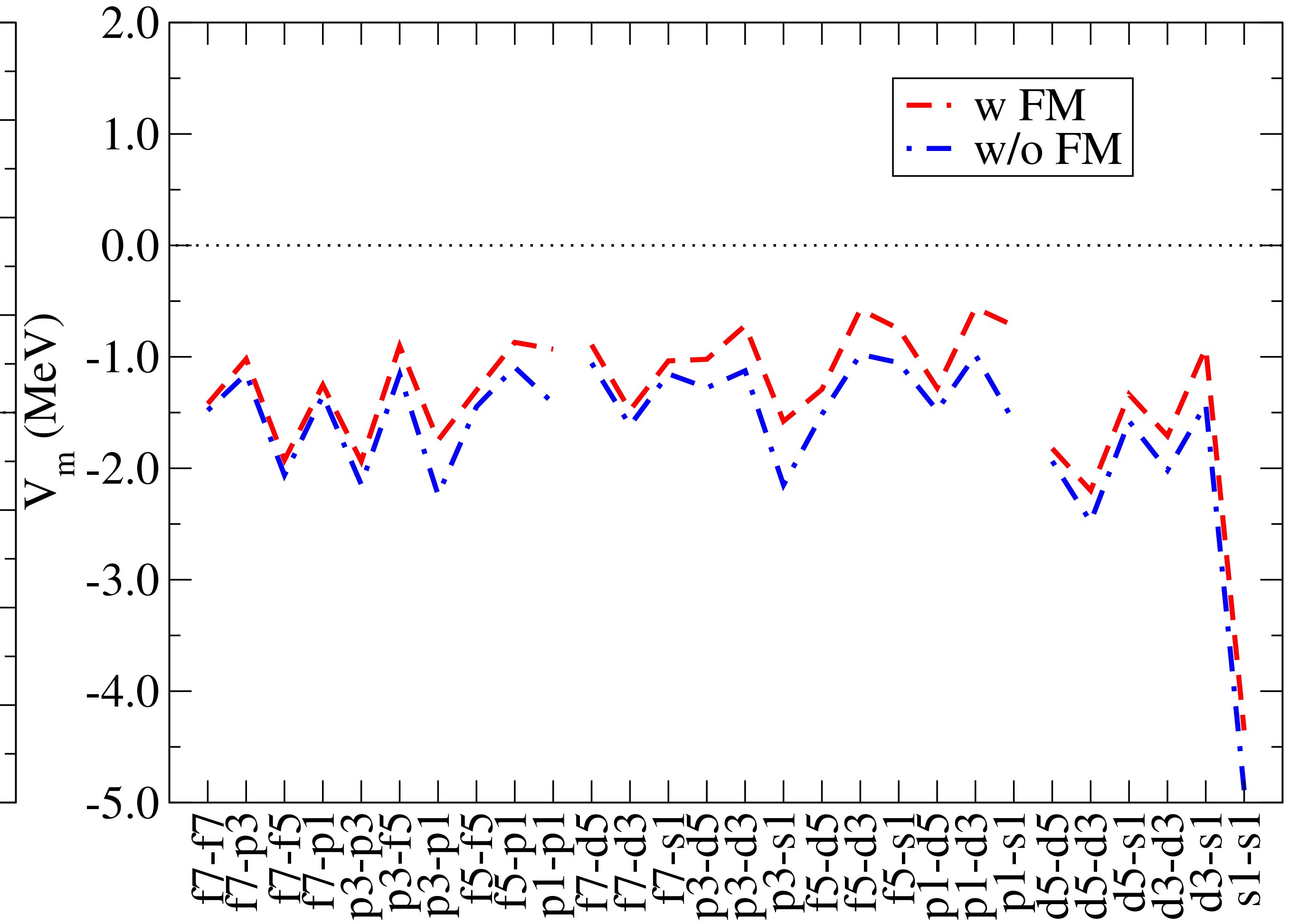
- Adding up effective 2N interaction derived from 3N interaction to EKK 2N effective interaction
- This is one of the lowest order interaction from 3N force and for higher order we are working on…

[1] T. Otsuka, T. Suzuki, J. D. Holt, A. Schwenk, and Y. Akaishi, Phys. Rev. Lett. 105, 032501 (2010).

sdpf-shell

Monopole interactions





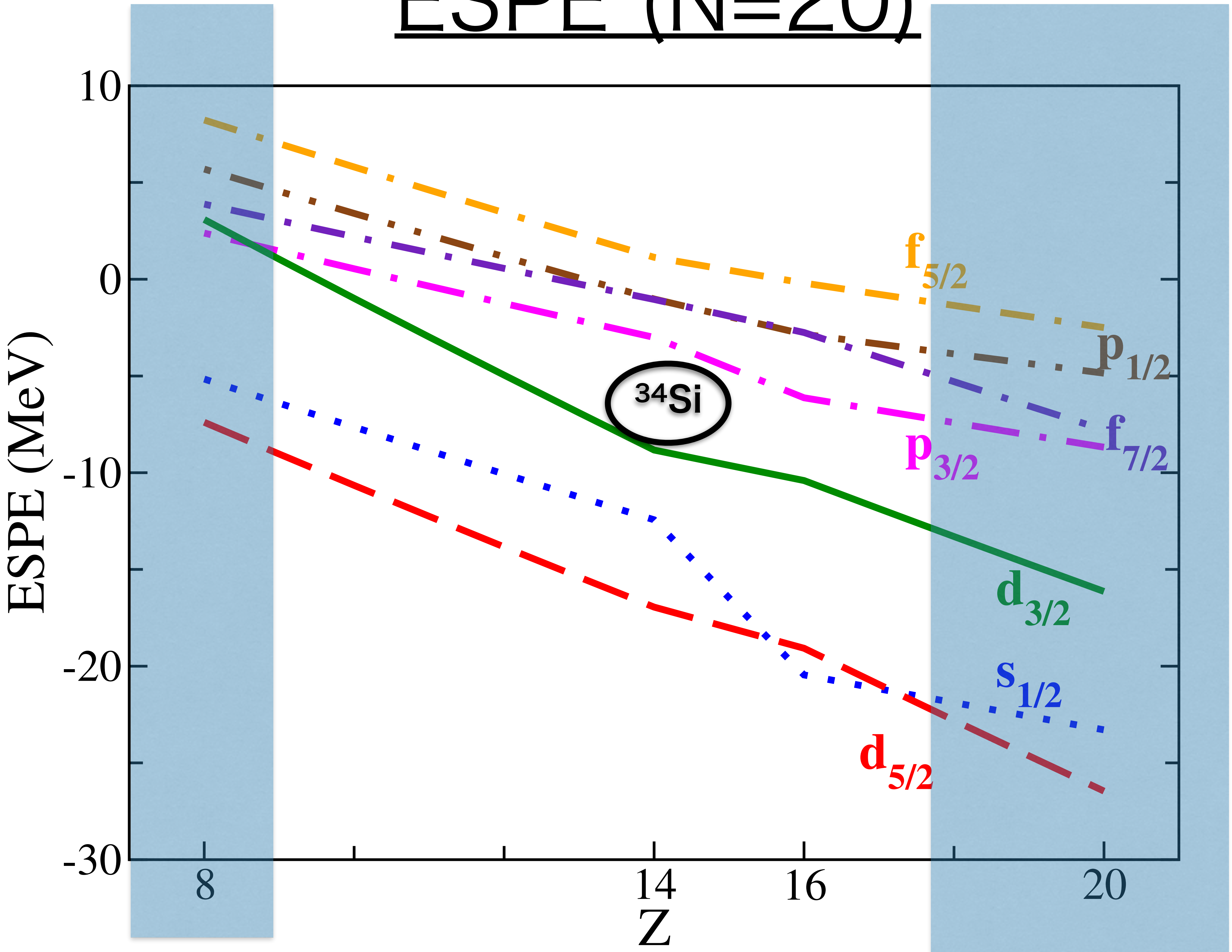
repulsive 3N force

SPE fitted

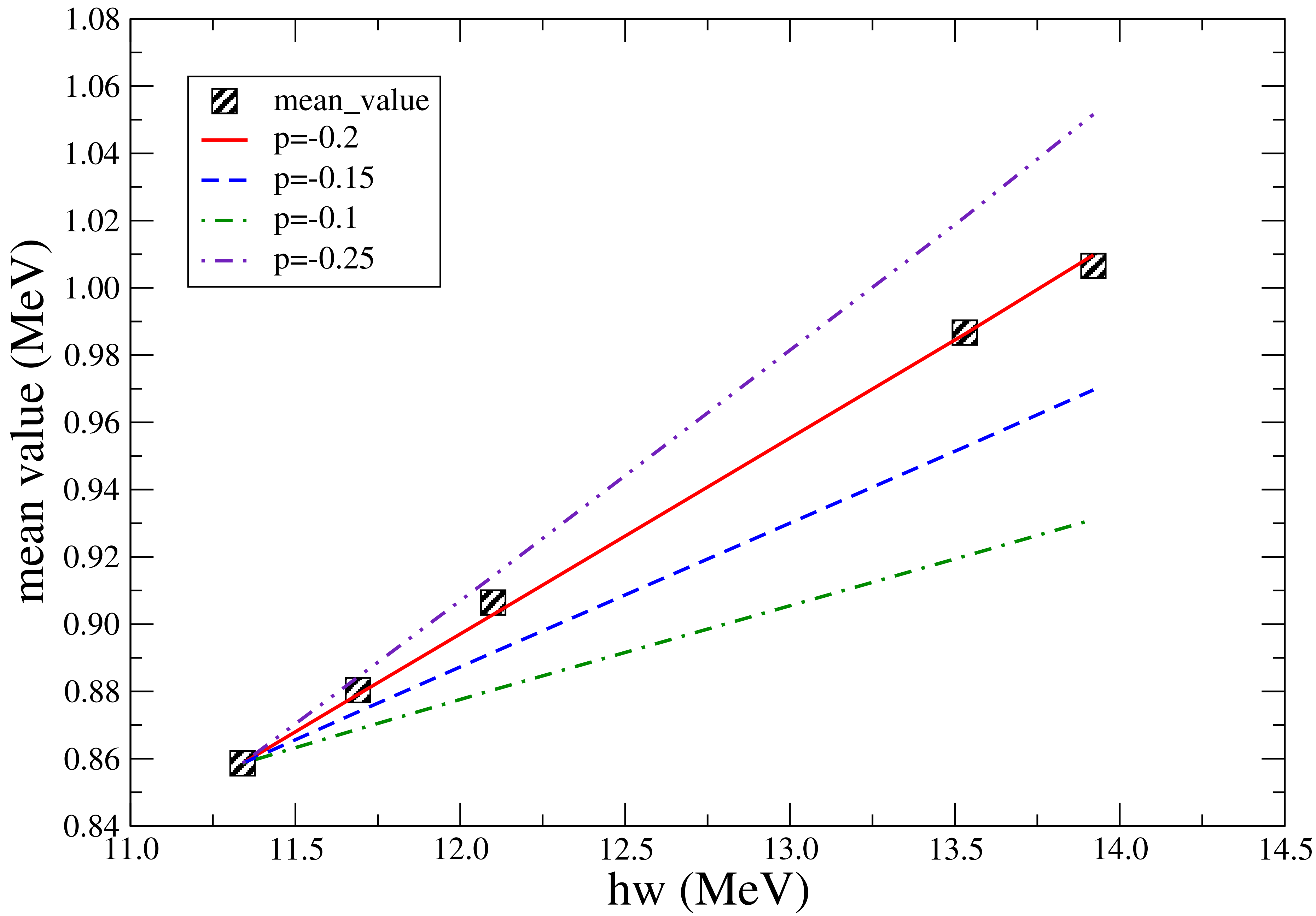
SPE set (MeV)

d5/2	-5.7
s1/2	-3.0
d3/2	1.8
f7/2	2.9
p3/2	3.6
p1/2	5.4
f5/2	5.4

ESPE (N=20)



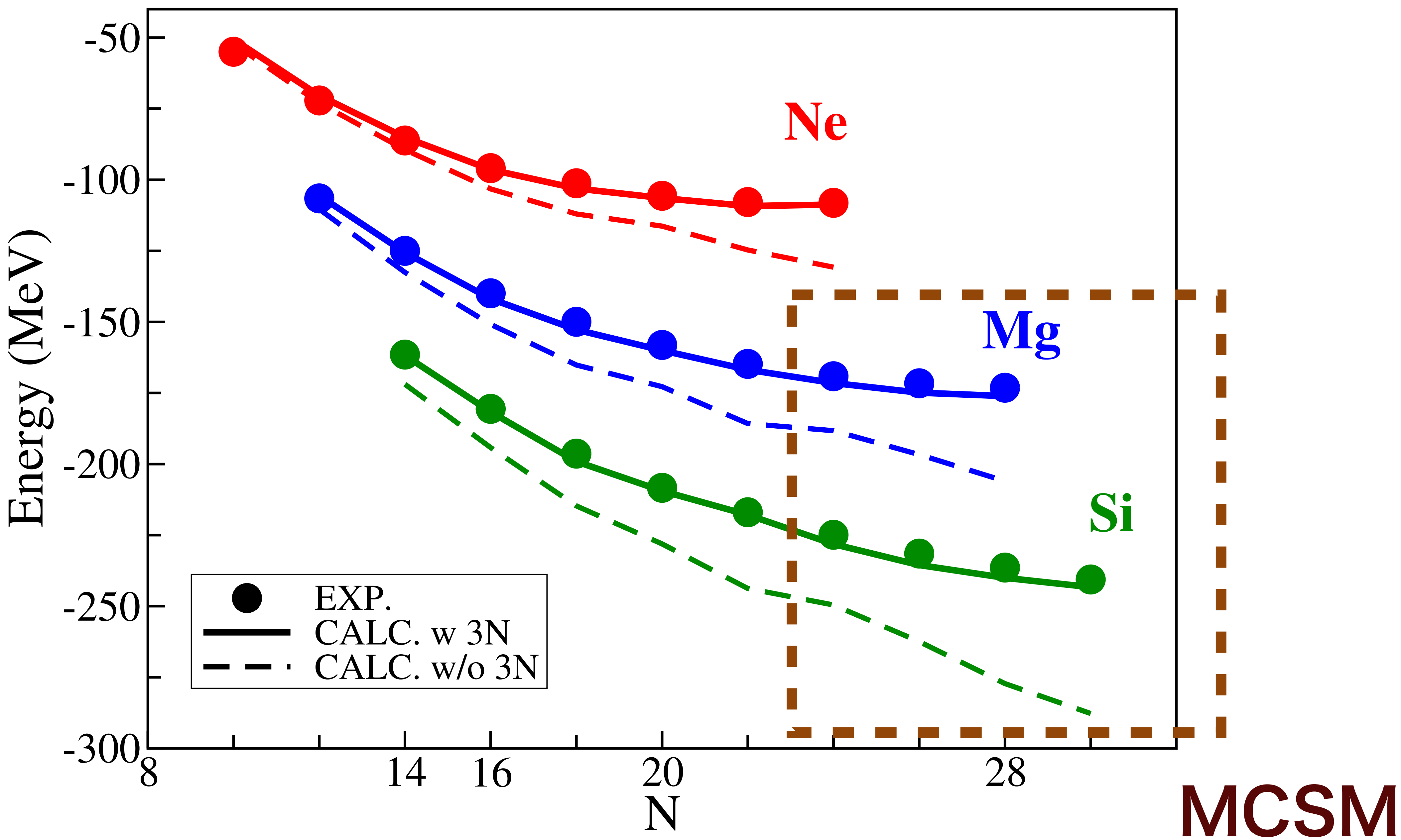
A-dependence of the TBMEs



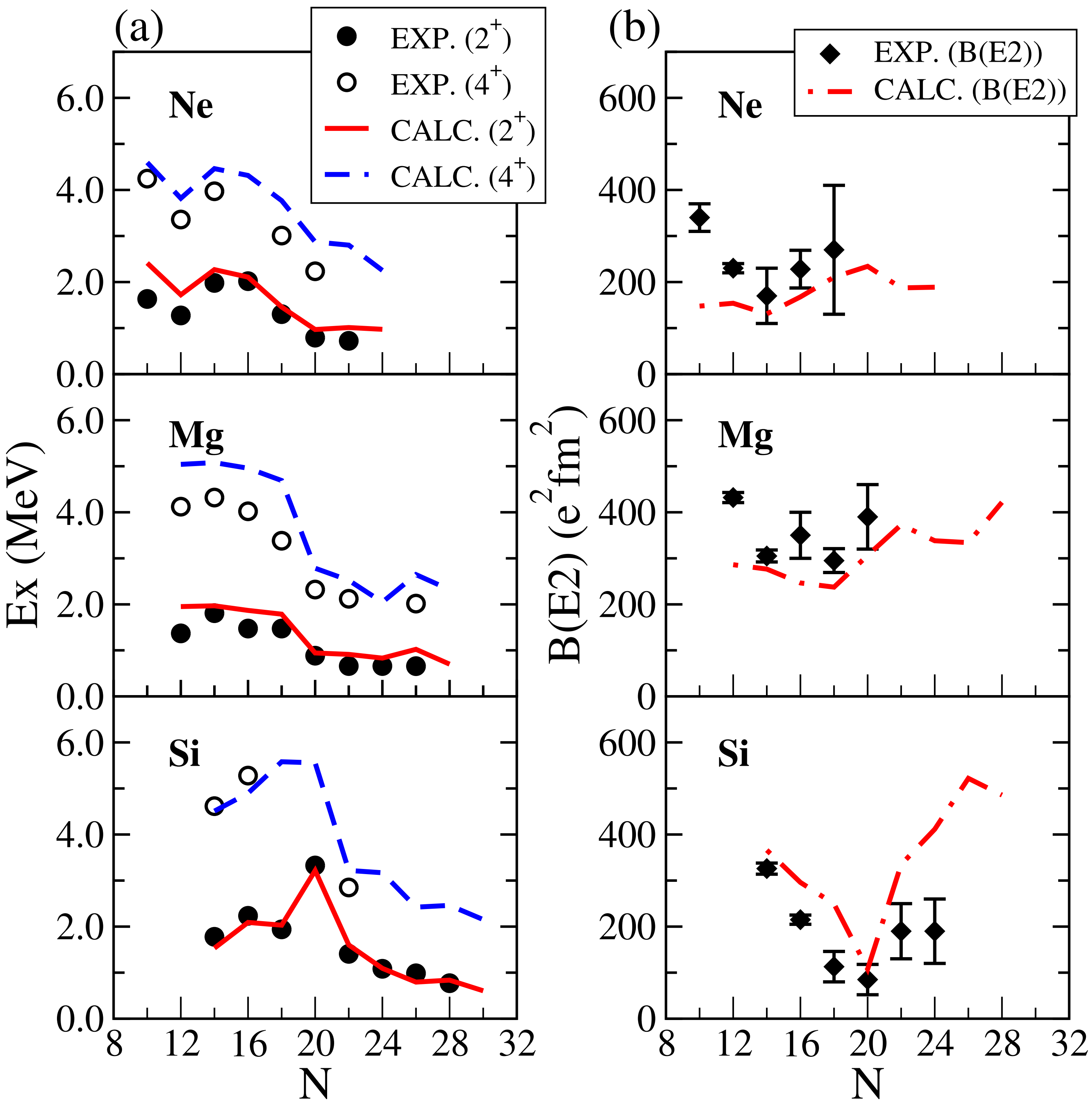
A-deps

(A/A_0)^p

Ground state energies



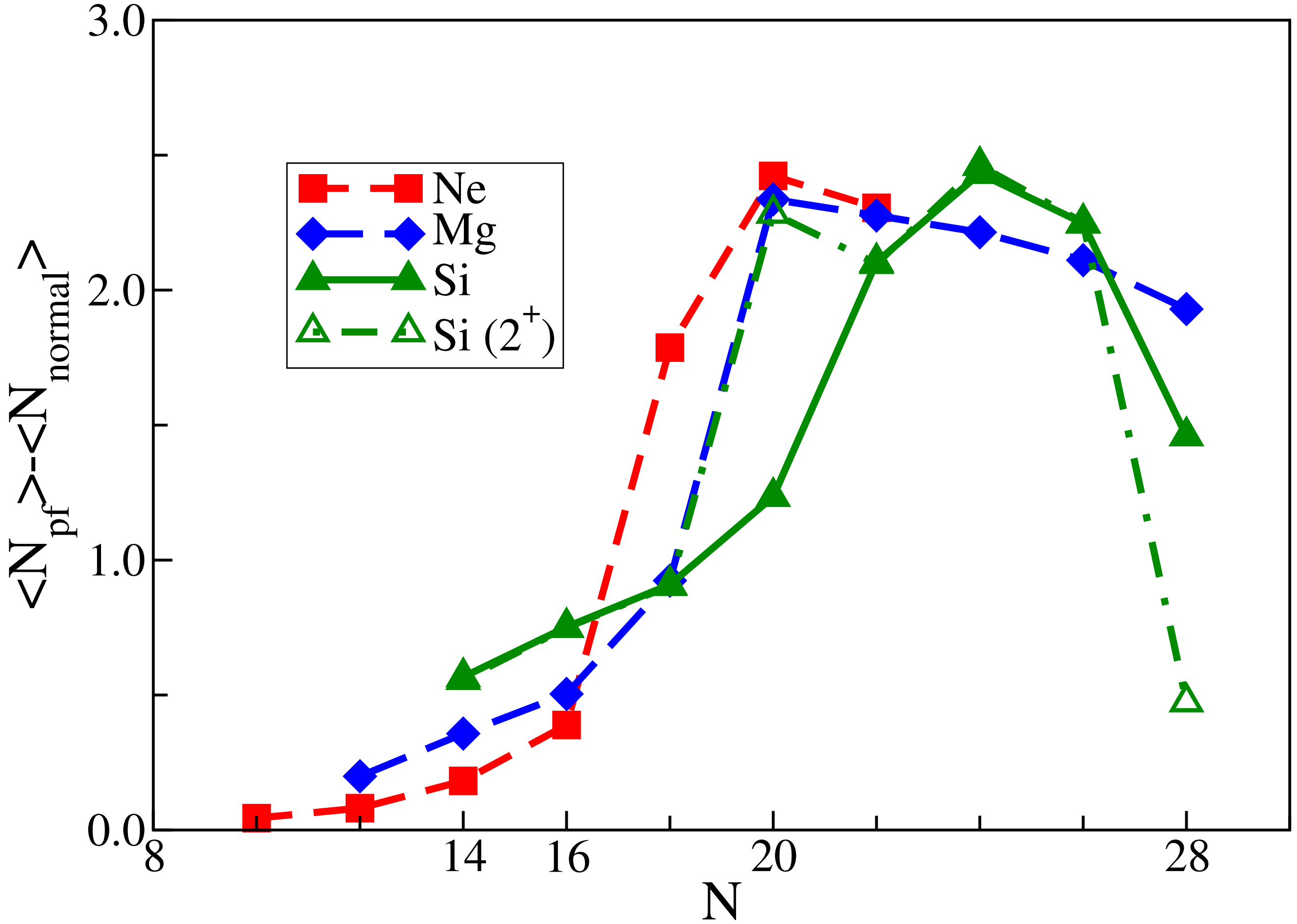
E2 and B(E2)



Clear indication of island of inversion

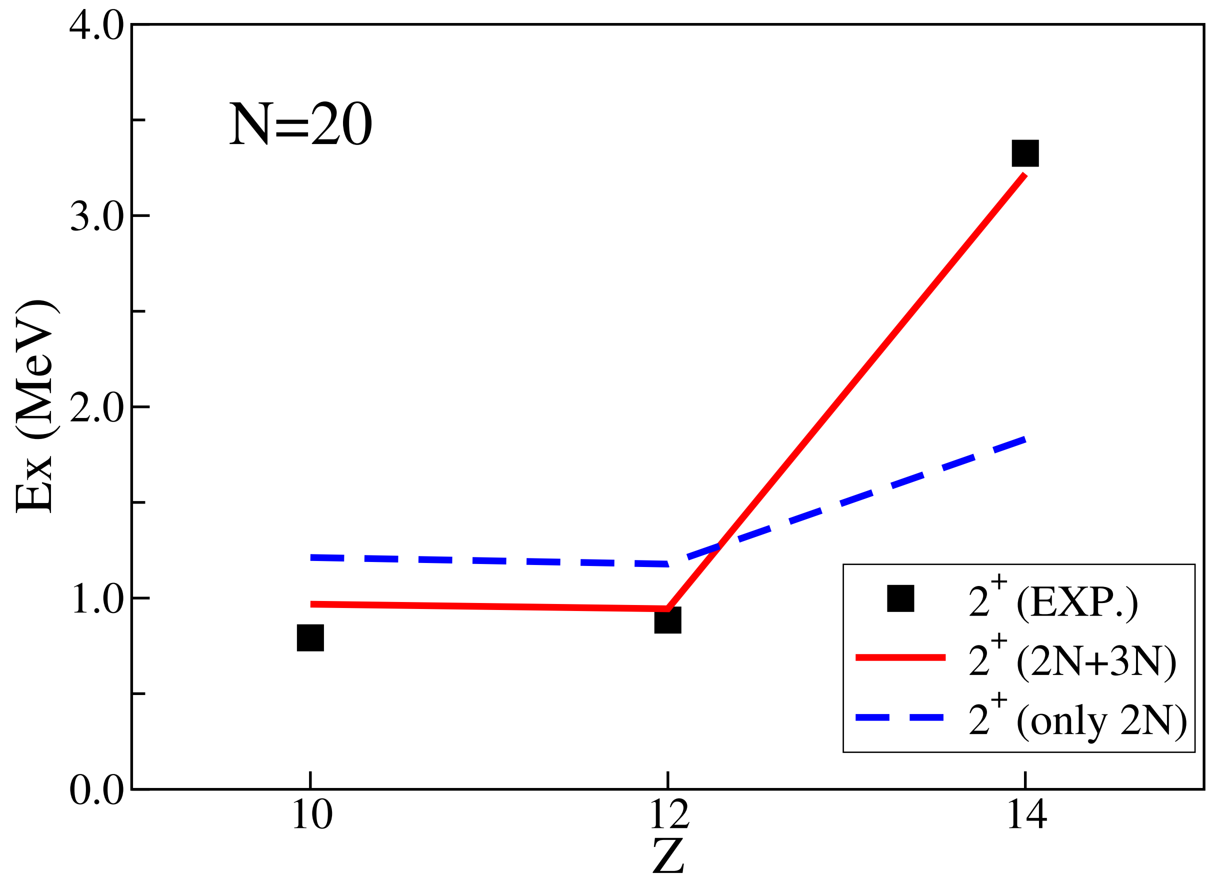
Effective charges (ep,en)=(1.2, 0.25)

island of inversion and 3N



Clear indication of island of inversion

Three body force drive the N=20 gap at ^{34}Si



Ca isotopes

Application to Calcium isotopes

setups

model space: full pfsdg-shell (2hw excitation)

N3LO (Vlowk 2.0 fm^{-1})

MBPT up to 3rd order

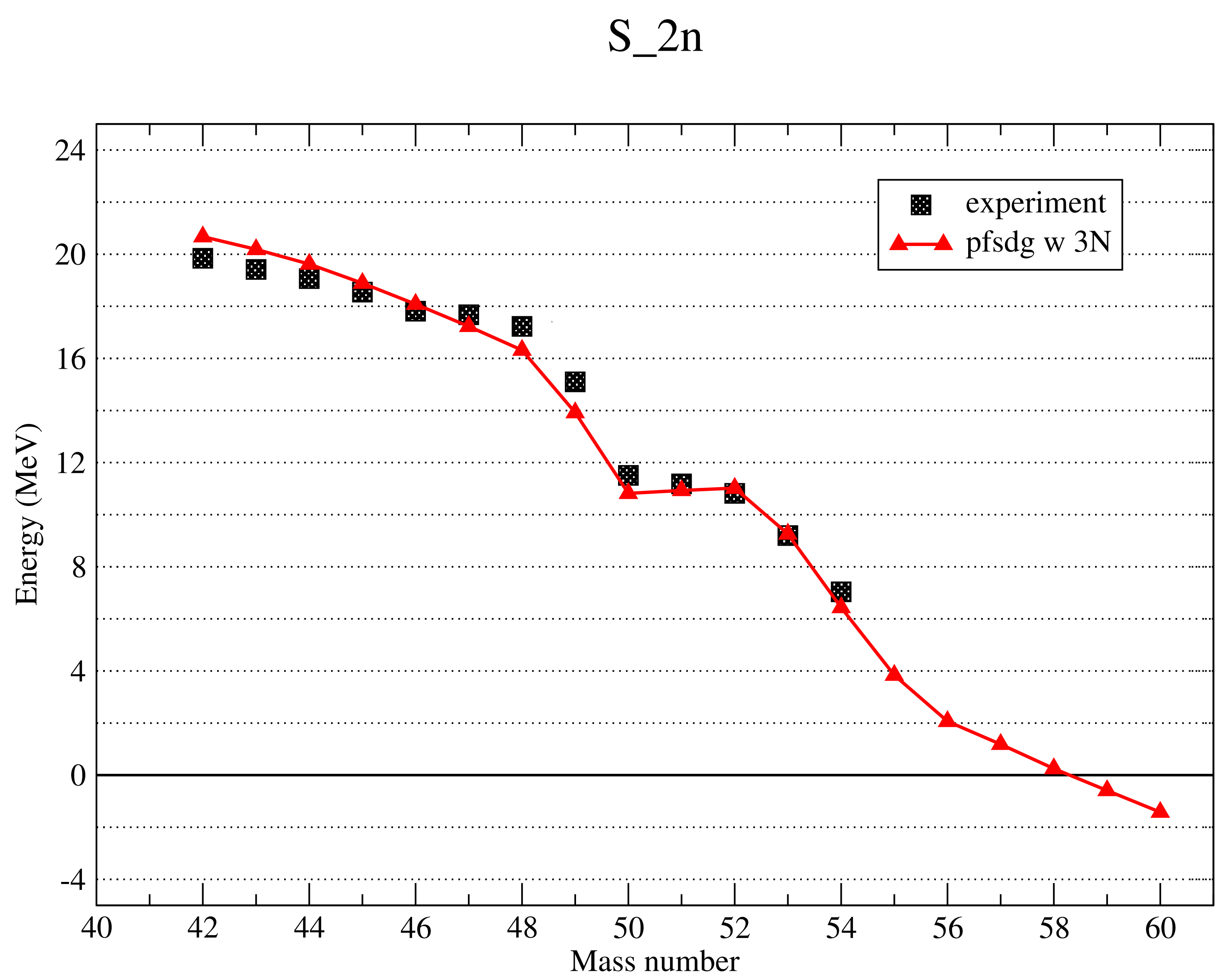
P+Q space: 17 hw

w and w/o 3N force

SPE modified

SPE set (MeV)

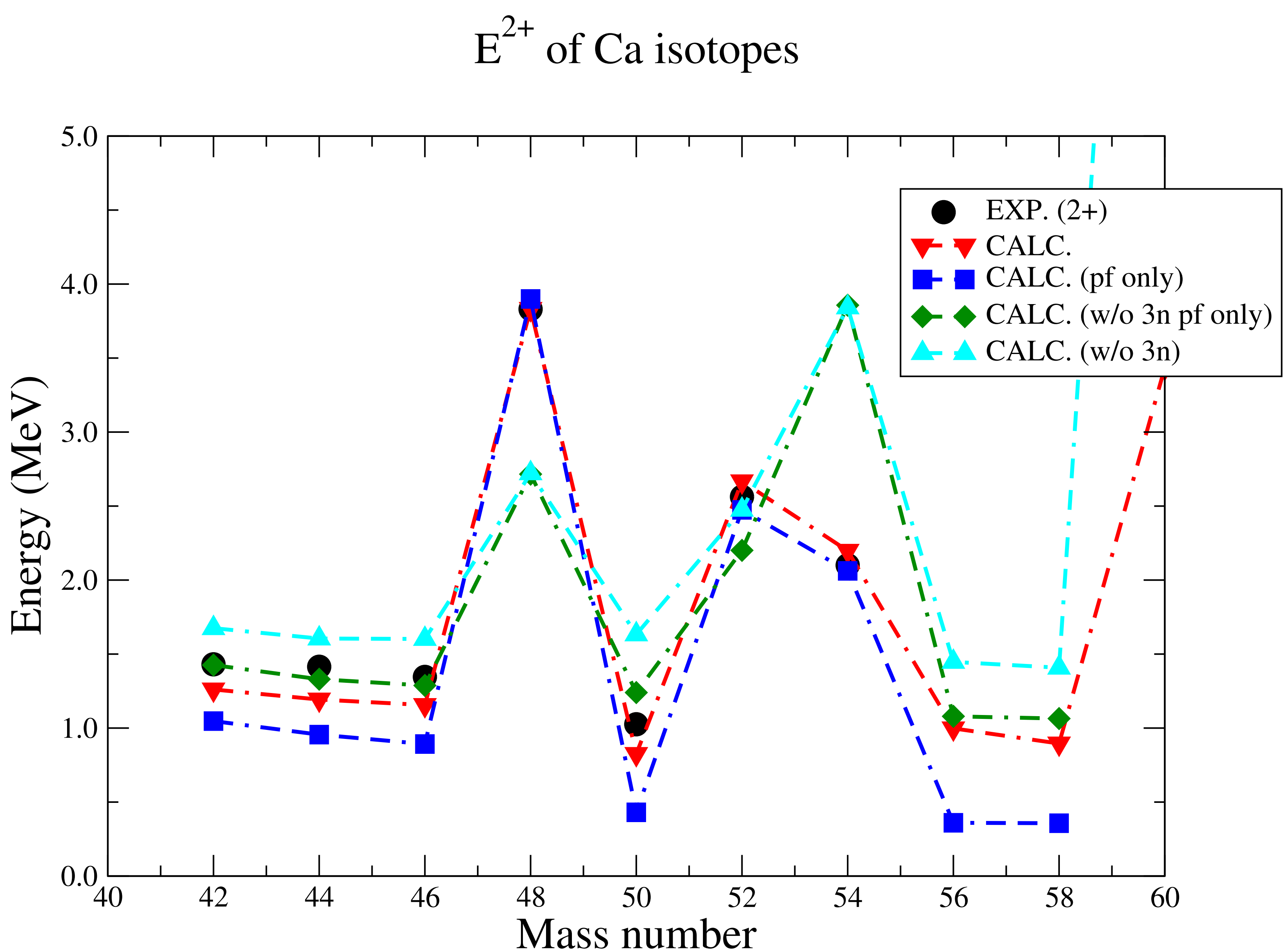
f7/2	-9.24	g9/2	0.0
p3/2	-5.44	g7/2	7.1
f5/2	-2.14	d5/2	1.8
p1/2	-2.94	d3/2	5.3
		s1/2	3.6

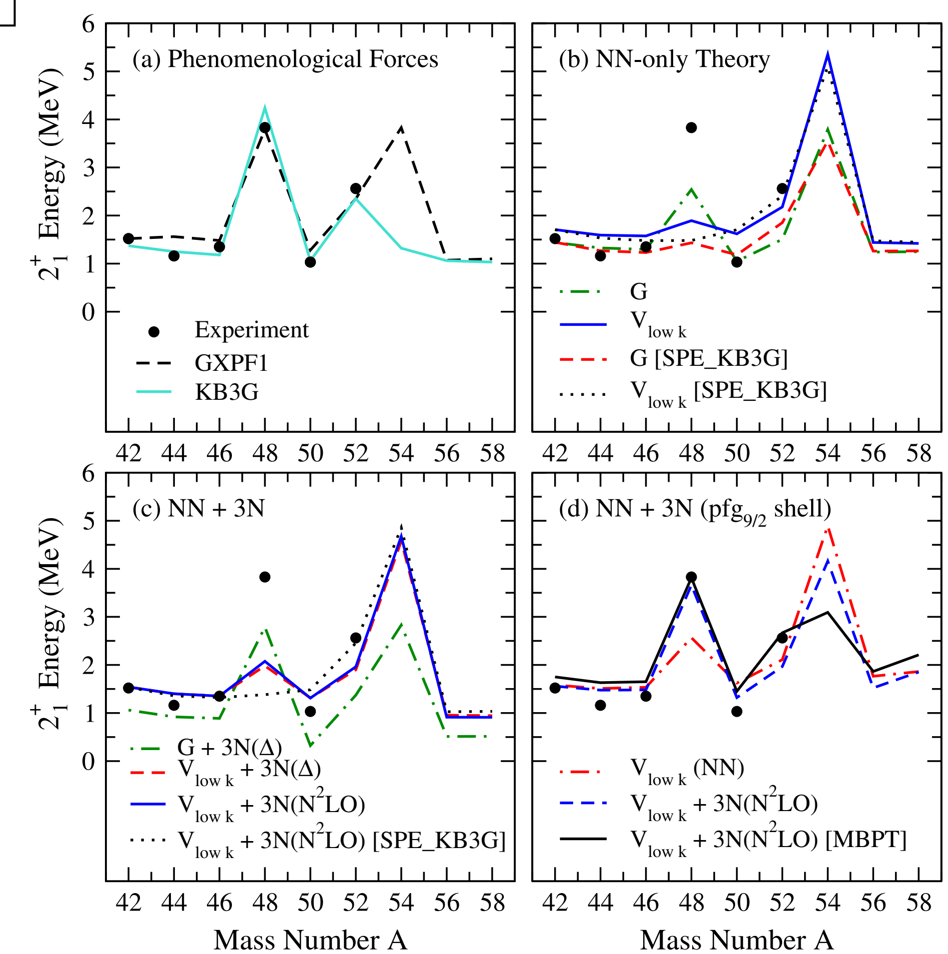


^{58}Ca slightly bound?

^{51}Ti 9/2- and Woods-Saxon potential (still investigating)

Application to Calcium isotopes



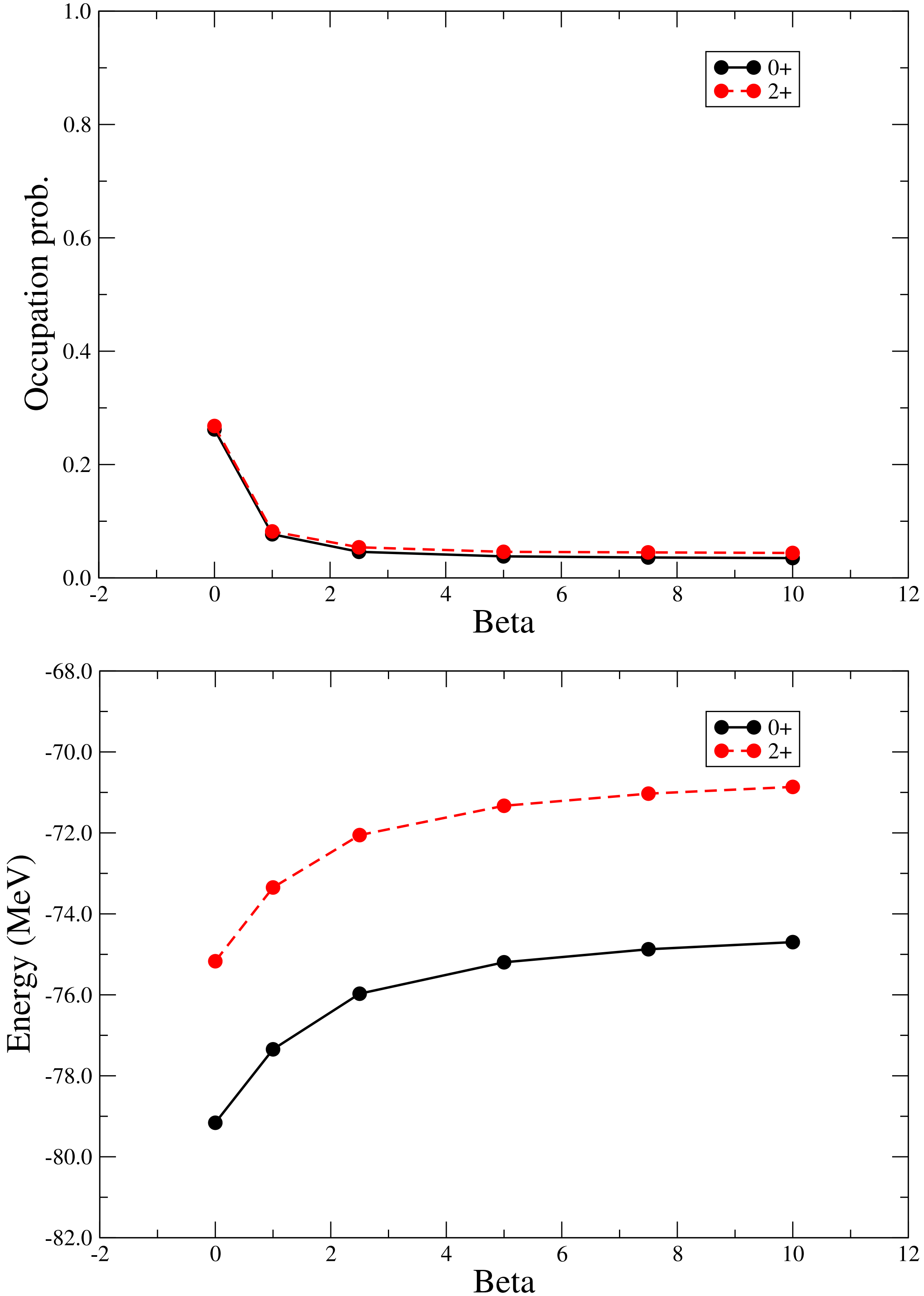


different observation?

maybe S.P.E and center of mass is different

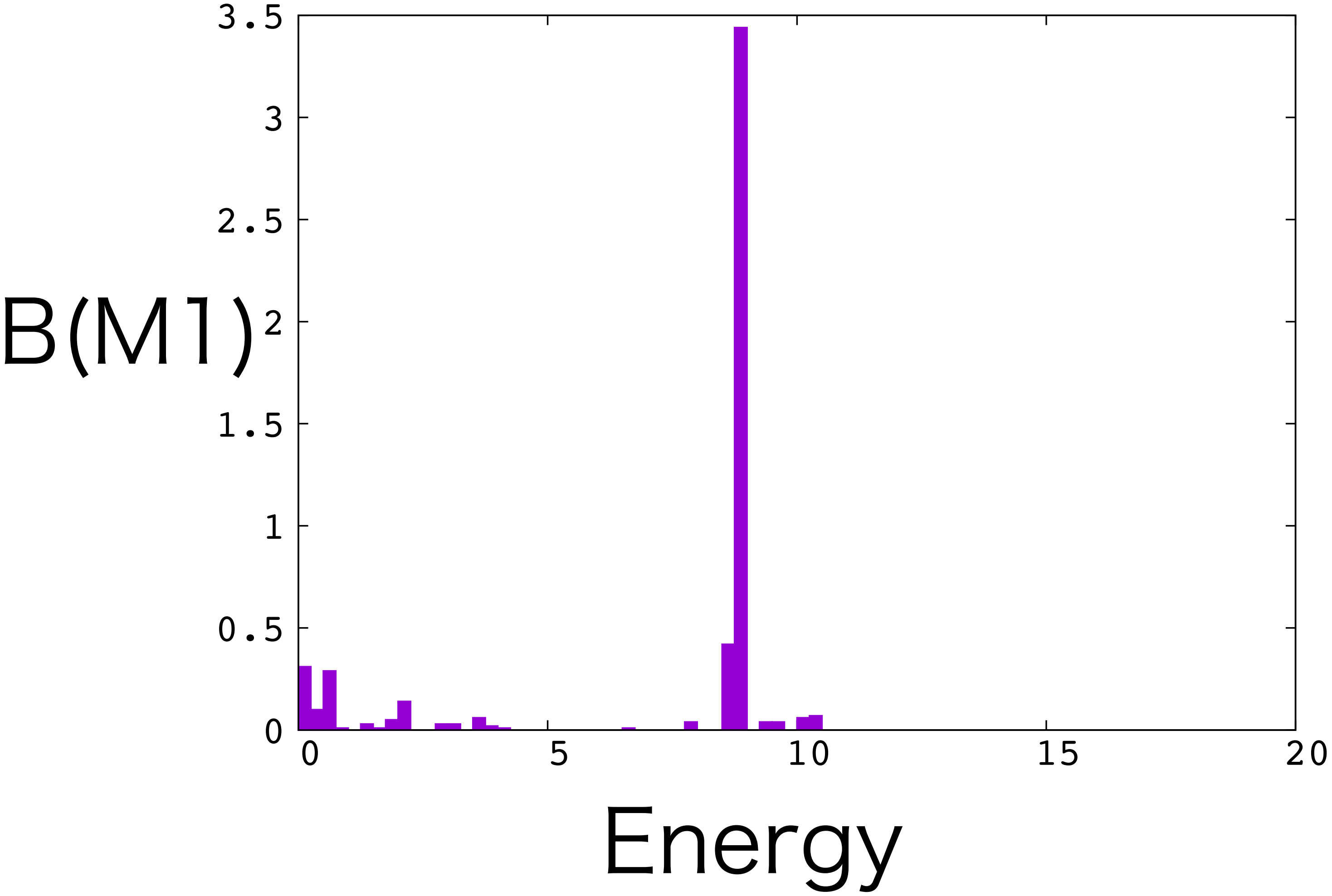
Lawson beta dependence

^{48}Ca



contribution beyond pf-shell almost vanish when beta>2.5

M1 transition of ^{48}Ca



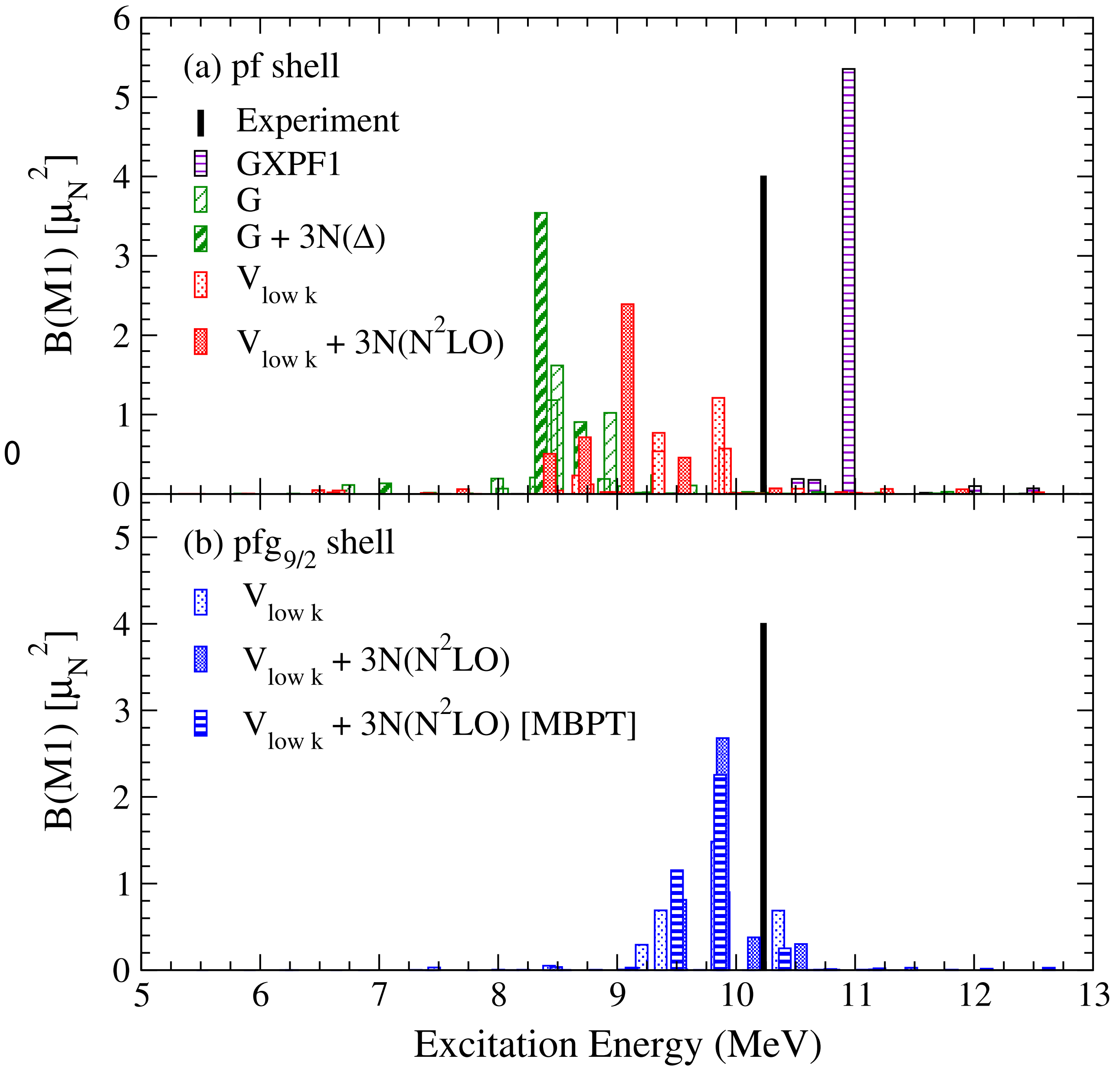
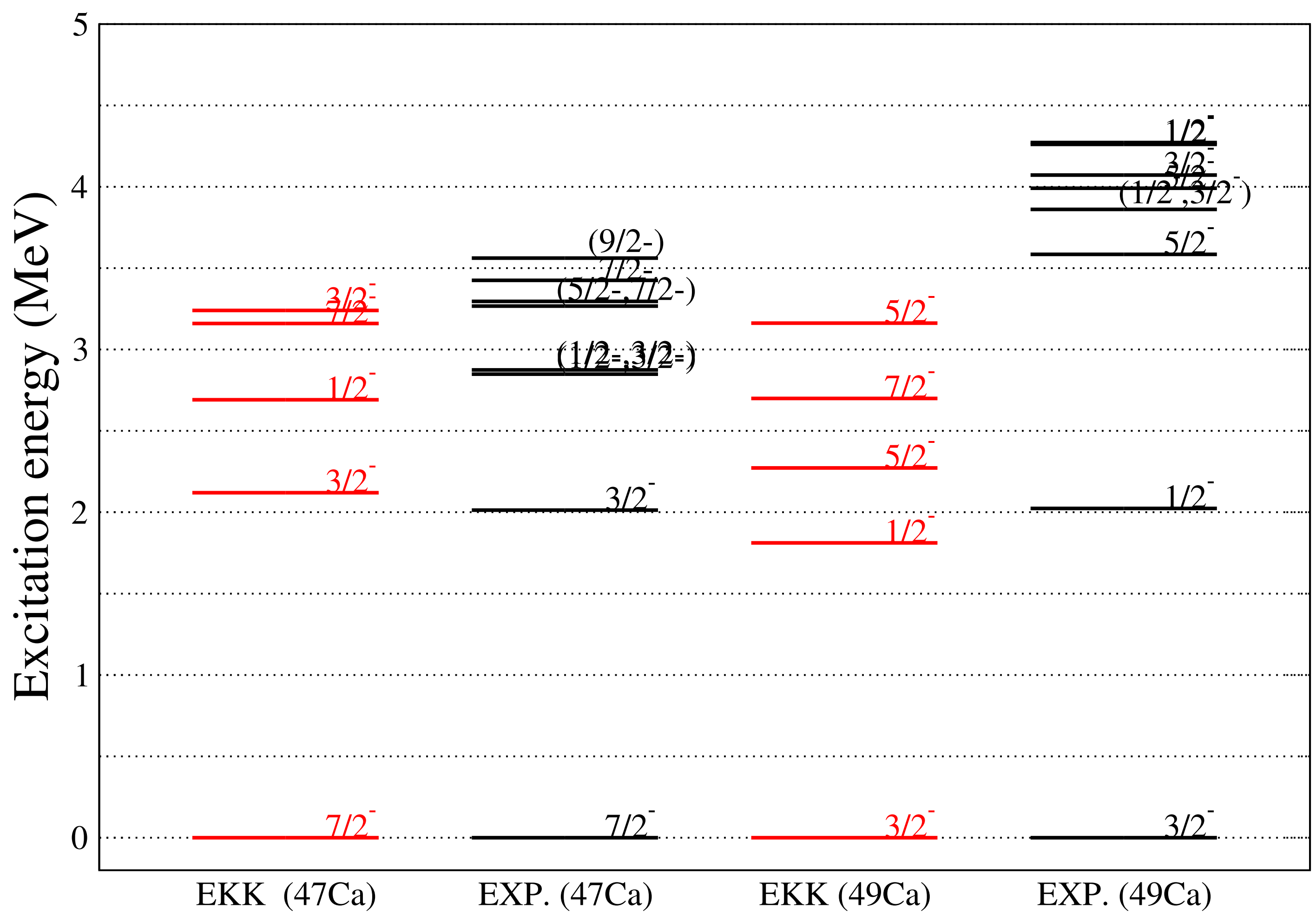


Figure from J. D. Holt, J. Menendez, J. Simonis, and A. Schwenk, Phys. Rev. C 90, 024312 (2014).

Odd isotopes



cf.

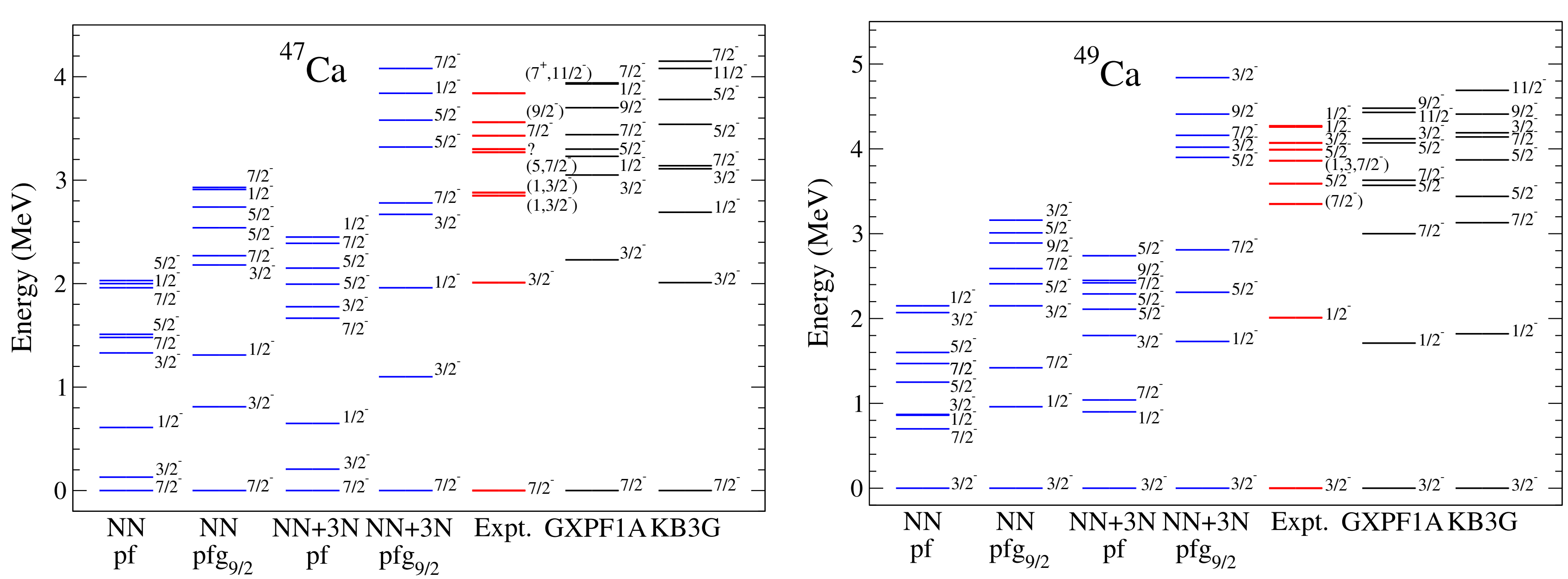
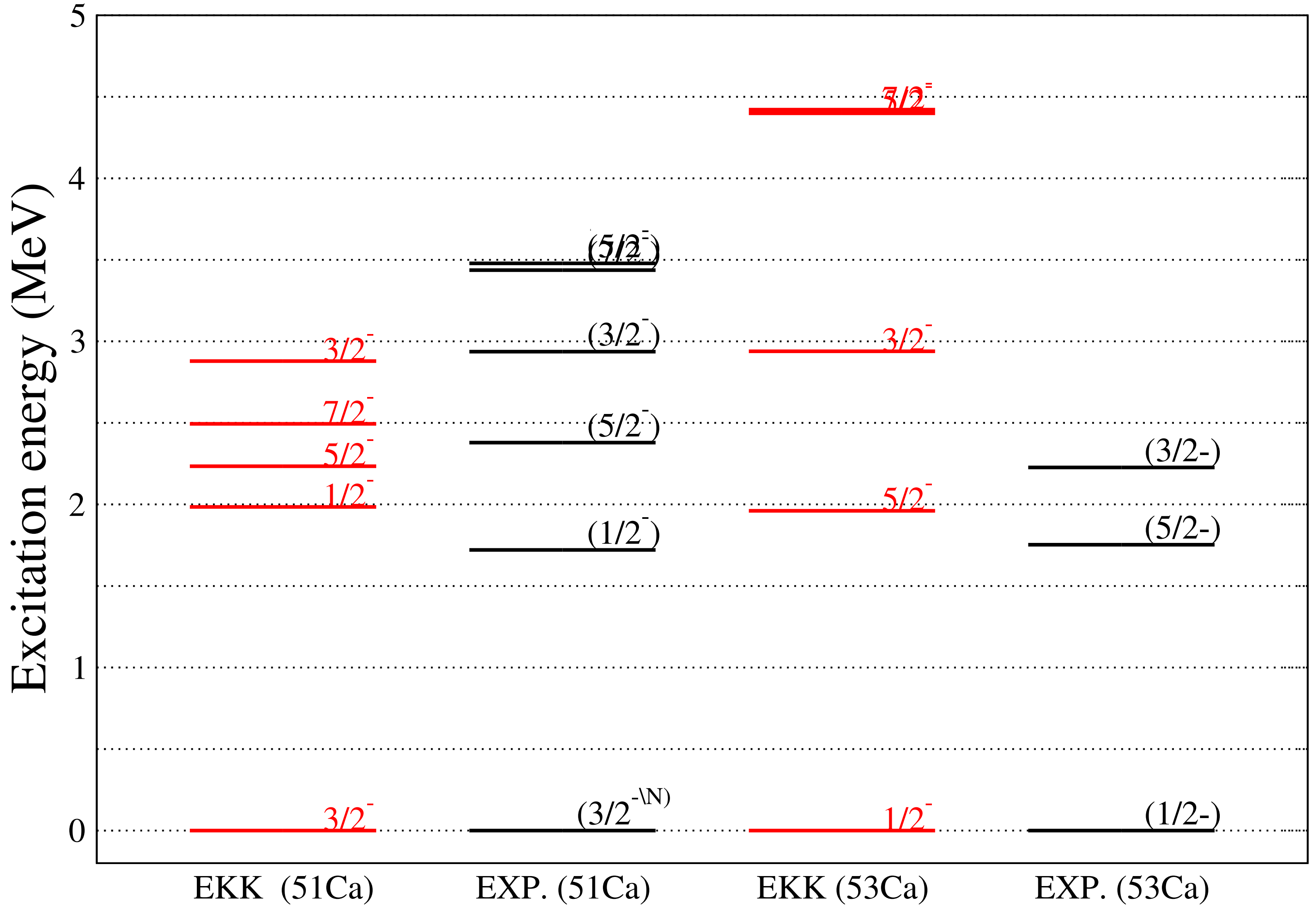


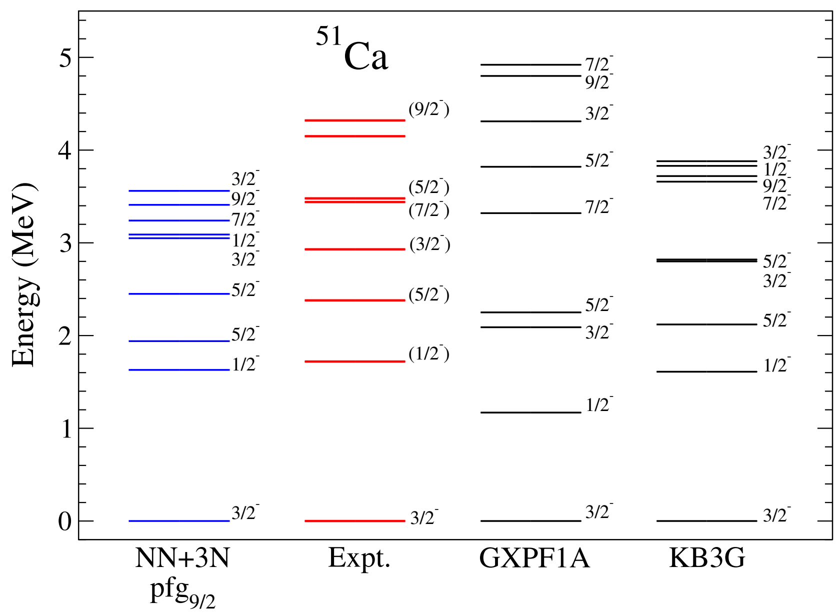
Figure from J. D. Holt, J. Menendez, J. Simonis, and A. Schwenk, Phys. Rev. C 90, 024312 (2014).

Odd isotopes



EXP old?

cf.



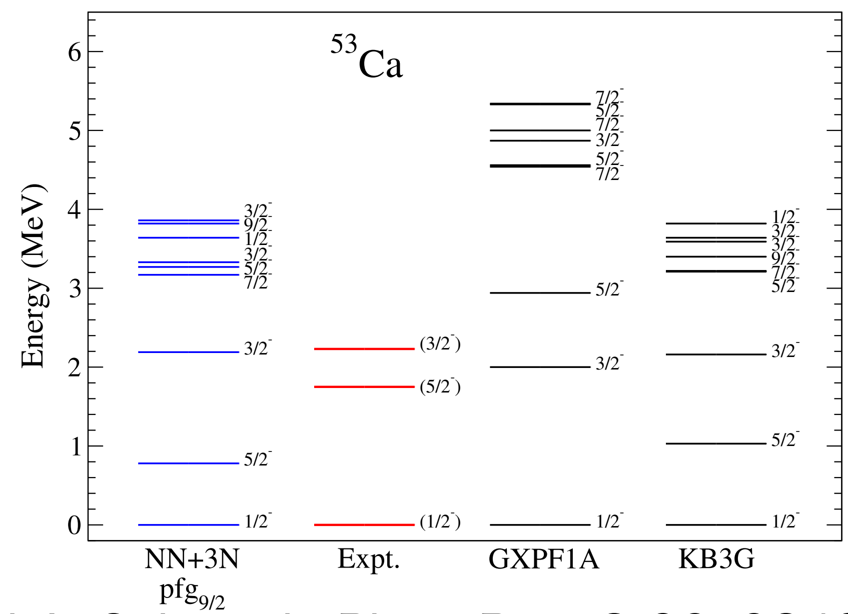


Figure from J. D. Holt, J. Menendez, J. Simonis, and A. Schwenk, Phys. Rev. C 90, 024312 (2014).

Summary and conclusion

- Introduced EKK method to derive the effective interaction for the shell model which is applicable to multi-shell system.
- As the first application of EKK method, Ca isotopes and island inversion in sdpf-shell is discussed.
- island of inversion is well described
- Ca isotopes need some more investigation

Collaborators

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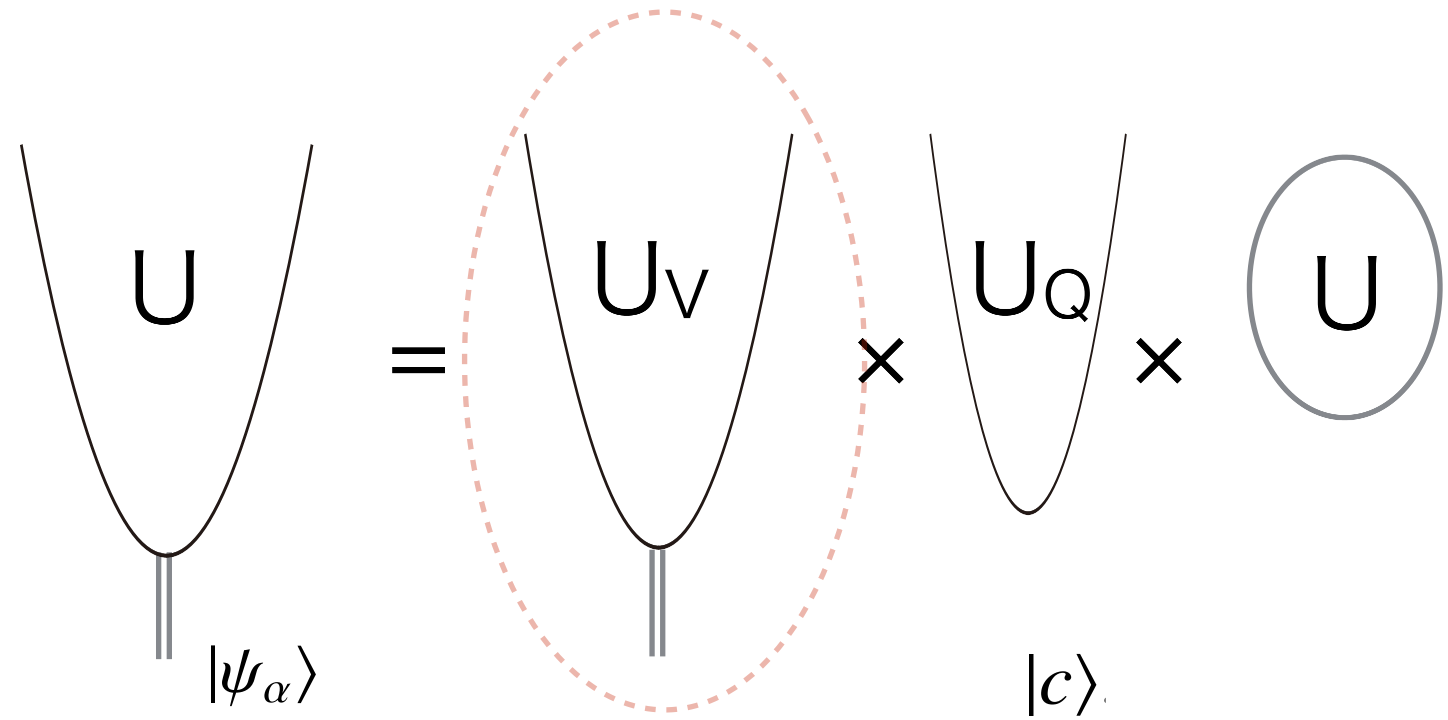
Factorization and folded diagram method (KK) 1/2

$$U(0,-\infty)|\psi_\alpha\rangle = U_V(0,-\infty)a_i^\dagger a_j^\dagger|c\rangle \times U(0,-\infty)|c\rangle,$$

$$U(0,-\infty)|c\rangle = U_Q(0,-\infty)|c\rangle \times \langle c|U(0,-\infty)|c\rangle,$$

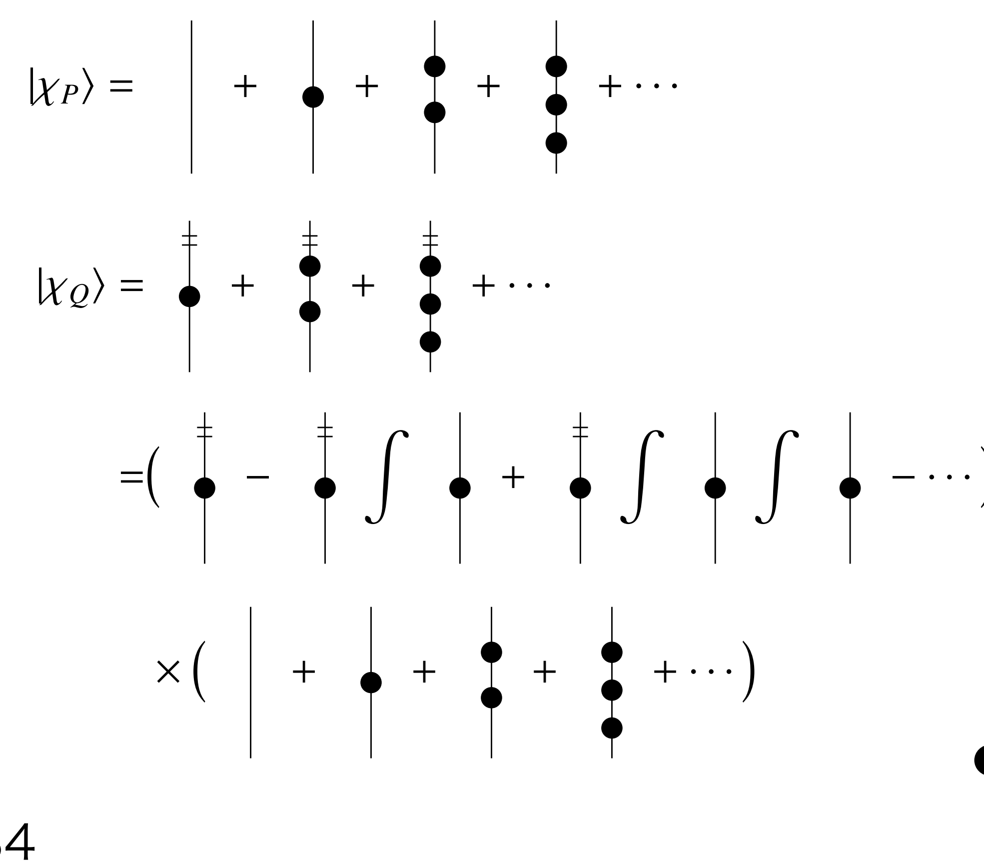
V: Valence linked
Q: terminate as Q-space state
C: core state



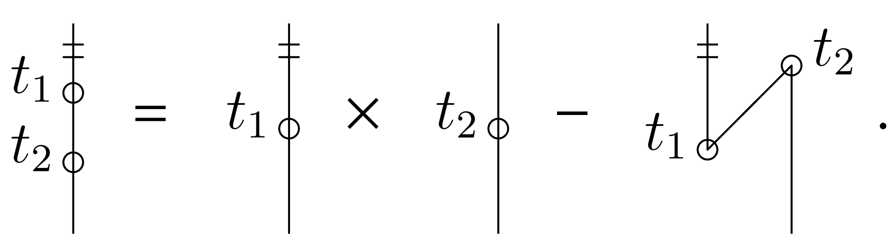
$$U_V(0,-\infty)|\psi_\alpha\rangle = |\chi_P\rangle + |\chi_Q\rangle.$$

$$U_V(0,-\infty)|\psi_\alpha\rangle = \sum_{\beta=1}^{D} U_{VQ}(0,-\infty)|\psi_\beta\rangle\langle\psi_\beta|U_V(0,-\infty)|\psi_\alpha\rangle.$$

P: terminate as P-space state
Q: terminate as Q-space state



folded diagram



Factorization and folded diagram method (KK) 2/2

Combining everything together,

$$U(0,-\infty)|\psi_\alpha\rangle = U_Q(0,-\infty)|c\rangle\langle c|U(0,-\infty)|c\rangle \times \sum_{\beta=1}^{d} U_{VQ}(0,-\infty)|\psi_\beta\rangle\langle\psi_\beta|U_V(0,-\infty)|\psi_\alpha\rangle$$

Energy of the core

Effective interaction

$$\rightarrow \sum_{\gamma=1}^{d} b_\gamma^\lambda H U_Q(0,-\infty)|c\rangle U_{VQ}(0,-\infty)|\psi_\gamma\rangle = \sum_{\delta=1}^{d} b_\delta^\lambda E_\lambda U_Q(0,-\infty)|c\rangle U_{VQ}(0,-\infty)|\psi_\gamma\rangle$$

$$b_\gamma^{(\lambda)} = \sum_{\alpha=1}^{d} C_\alpha^{(\lambda)} \frac{\langle\psi_\gamma|U_V(0,-\infty)|\psi_\alpha\rangle\langle c|U(0,-\infty)|c\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}$$

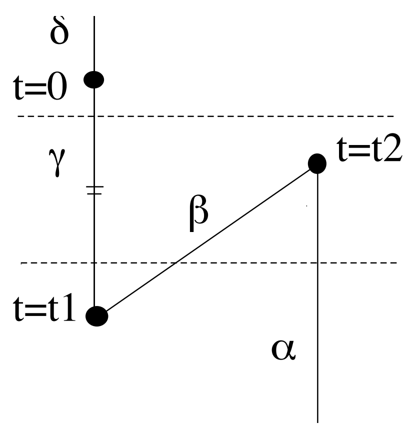
Divergences are canceled out !

Effective interaction Veff include Q-box and its infinite order repetition

$$V_{\rm eff} = \hat{Q}(\epsilon_0) - \hat{Q}'(\epsilon_0)\int \hat{Q}(\epsilon_0) + \hat{Q}'(\epsilon_0)\int \hat{Q}(\epsilon_0)\int \hat{Q}(\epsilon_0)\cdots$$

$$\begin{aligned}\hat{Q}(E) &= PVP + PVQ\frac{1}{E-QHQ}QVP \\ &= PVP + PVQ\frac{1}{E-QH_0Q}QVP + PVQ\frac{1}{E-QH_0Q}QVQ\frac{1}{E-QH_0Q}QVP + \cdots\end{aligned}$$

Folded diagram and energy derivative



$$=\frac{V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}}{(\epsilon_\alpha-\epsilon_\gamma-(\epsilon_\alpha-\epsilon_\beta))(\epsilon_\alpha-\epsilon_\gamma)}$$

$$=V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}\frac{\Big((\epsilon_\alpha-\epsilon_\gamma)-(\epsilon_\alpha-\epsilon_\beta)\Big)^{-1}-(\epsilon_\alpha-\epsilon_\gamma)^{-1}}{\epsilon_\alpha-\epsilon_\beta}$$

in the limit of $\epsilon_\beta \to \epsilon_\alpha$

$$=\frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{V_{\beta\gamma}V_{\gamma\delta}}{\omega-\epsilon_\gamma}\right)_{\omega=\alpha}\times V_{\alpha\beta}$$

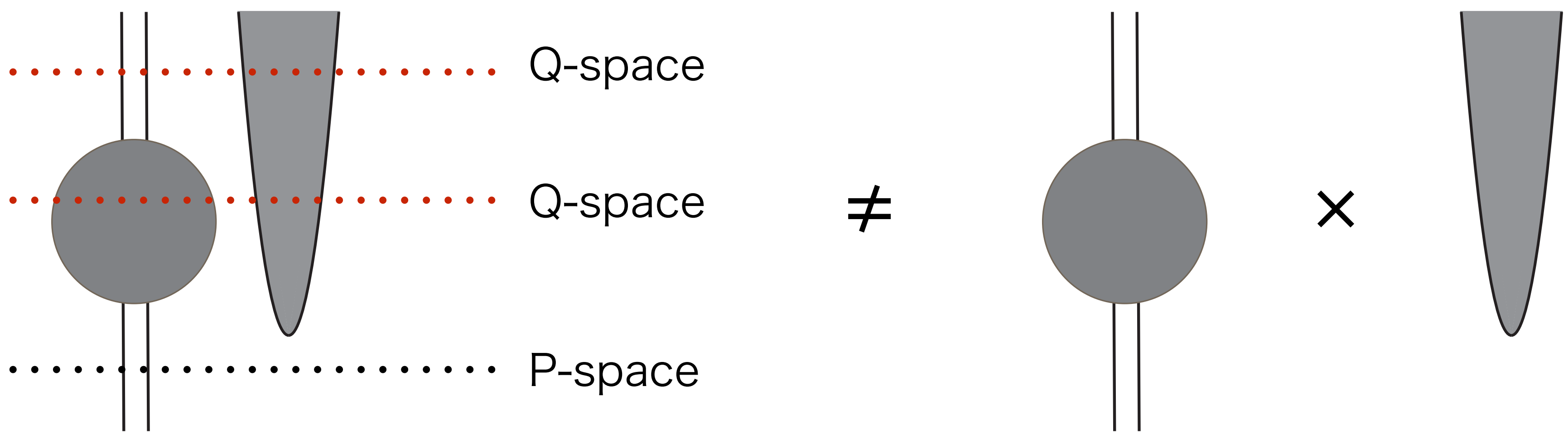
→ Folded diagrams can be calculated by energy derivative if the model space is degenerate

Final expression of the V_{eff}

$$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty}\hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

Factorization theorem in EKK method

Factorization theorem does not hold in EKK method naively



$$
\begin{aligned}
H &= H_0 + V \\
&= H_0' + V' \\
&= H_0' - P(E - H_0)P + V \\
&= H_0' + V_1 + V,
\end{aligned}
$$

Insert V_1 vertex up to infinite order