



NEW ADVANCES IN SHELL MODEL CALCULATIONS: APPLICATIONS AROUND DOUBLY MAGIC NUCLEI ^{40}Ca AND ^{132}Sn

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ICNT Program: Theory for open-shell near the limits of stability,
May 11-29, 2015

Outline

PART 1 : AROUND ^{40}Ca

1. Ca isotopes shift
2. Isomer shift in ^{38}K
3. Effect of pairing correlations

PART 2 : AROUND ^{132}Sn

1. Choose Valence space/Core
2. develop an effective interaction
3. Calculation of the energies, B(E2) and masses in $^{134,136,138}Sn$,
 - ▶ Effect of core excitations
 - ▶ Closure or no of the sub-shell at N=90
4. Other applications to the open n-p systems : Te, Xe, Ba, Ce and Nd.

PART I : ISOMER SHIFT IN ^{38}K

REMINDER ON Ca ISOTOPES SHIFT



13 December 2001

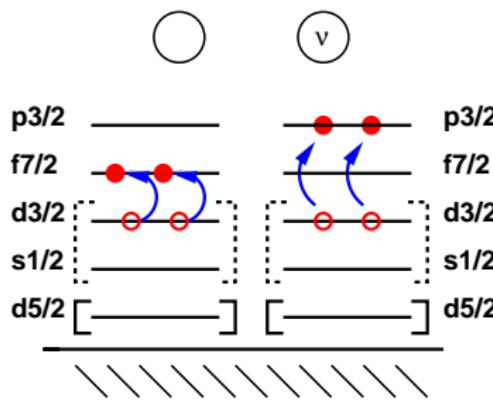
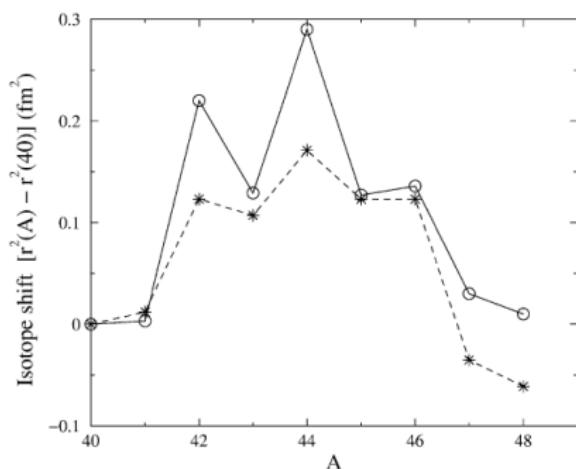
PHYSICS LETTERS B

Physics Letters B 522 (2001) 240–244

www.elsevier.com/locate/npe

Shell model description of isotope shifts in calcium

E. Caurier ^a, K. Langanke ^b, G. Martínez-Pinedo ^{b,c}, F. Nowacki ^d, P. Vogel ^e



REMINDER ON Ca ISOTOPES SHIFT



13 December 2001

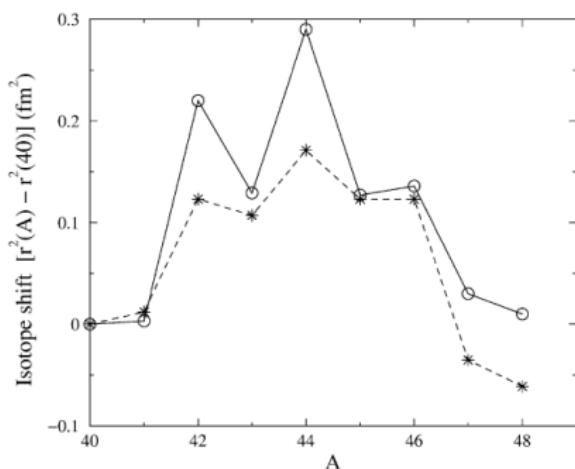
PHYSICS LETTERS B

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Shell model description of isotope shifts in calcium

E. Caurier ^a, K. Langanke ^b, G. Martínez-Pinedo ^{b,c}, F. Nowacki ^d, P. Vogel ^e



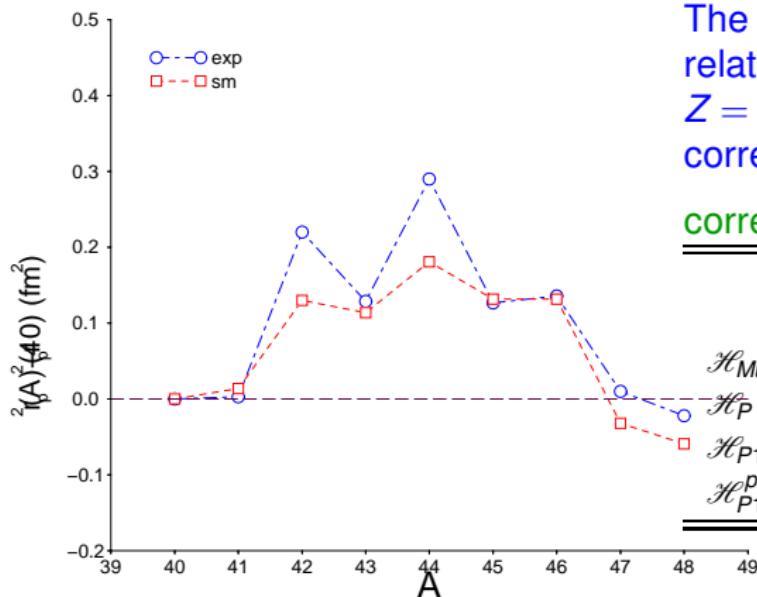
ZBM2 interaction :

- ▶ based on realistic TBME
- ▶ monopole corrections to ensure ^{40}Ca and ^{48}Ca gaps
- ▶ full space calculations
- ▶ almost free of center of mass contamination
- ▶ provides very good spectroscopy at *sd-pf* interface

Ca isotopes shift

$$\delta r_c^2 = \frac{1}{Z} n_{fp}^\pi(A) b^2 \left\{ \begin{array}{l} n_{fp}^\pi \\ b = 1.974 fm \end{array} \right. \begin{array}{l} \text{π occupation probability in fp shell} \\ \text{oscillateur parameter} \end{array}$$

Isotope shifts in Ca chain



The parabolic dependence on A is related to the partial breaking of the $Z = 20$ shell closure due to the $\pi\nu$ correlations

correlation energies

	^{40}Ca	^{42}Ca	^{44}Ca	^{46}Ca	^{48}Ca
$\mathcal{H}_{\text{Multi}}$	-13.1	-17.5	-18.1	-13.6	-6.2
\mathcal{H}_P	-9.7	-12.5	-11.9	-8.7	-4.0
\mathcal{H}_{P1}	-8.9	-11.8	-11.3	-8.4	-3.9
\mathcal{H}_{P1}^{pn}	-2.95	-2.2	-1.4	-0.8	-0.2

ISOMER SHIFT IN ^{38}K

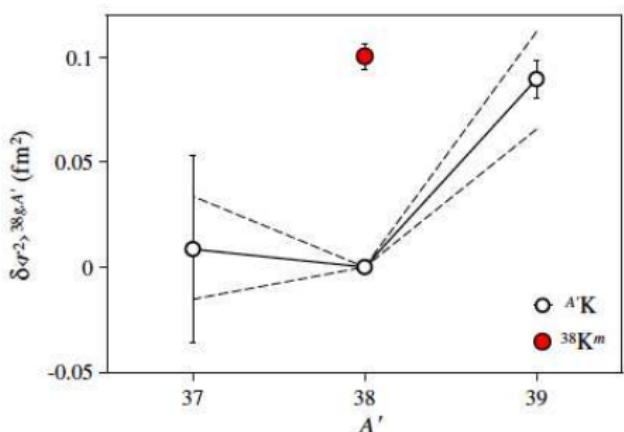
PRL 113, 052502 (2014)

PHYSICAL REVIEW LETTERS

week ending
1 AUGUST 2014

Proton-Neutron Pairing Correlations in the Self-Conjugate Nucleus ^{38}K Probed via a Direct Measurement of the Isomer Shift

M. L. Bissell,^{1,*} J. Papuga,¹ H. Naïdja,^{2,3,4} K. Kreim,⁵ K. Blaum,⁵ M. De Rydt,¹ R. F. Garcia Ruiz,¹ H. Heylen,¹ M. Kowalska,⁶ R. Neugart,^{5,7} G. Neyens,¹ W. Nörtershäuser,^{7,8} F. Nowacki,² M. M. Rajabali,¹ R. Sanchez,^{3,9} K. Sieja,² and D. T. Yordanov⁵



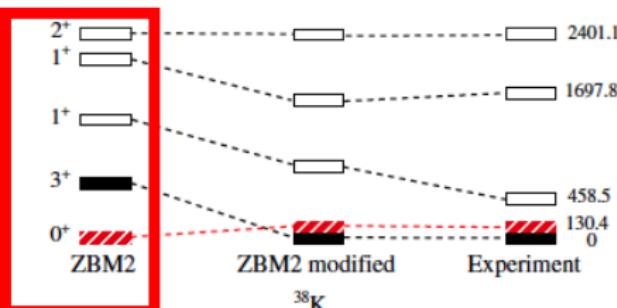
$$\delta\sigma_c^2(0^+) - \delta\sigma_c^2(3^+) = 0.1 \text{ fm}^2$$

	$3^+_1 GS$	0^-_m
\mathcal{H}_{Multi}	-7.9	-13.2
\mathcal{H}_P	-4.2	-9.7
\mathcal{H}_{P1}	-4.2	-9.2
\mathcal{H}_{P1}^{pn}	-1.4	-6.1

The strong neutron-proton correlations in 0^+ compared to 3^+

proton occupancy of fp shell is reduced in 3^+ compared to 0^+

SM RESULTS : ZBM2



✗ bad level scheme

	$n_{fp}^\pi(38m)$	$n_{fp}^\pi(38g)$	$\delta\langle r_c^2 \rangle^{38g,38m}$ (fm 2)
ZBM2	0.86	0.50	0.075
ZBM2 modified	0.82	0.41	0.085
Experiment			0.100(6)

✓ good variation of r_c

T=1 matrix elements of the ZBM2 are too strong with respect to the T=0, producing an inversion of the 0 $^+$ and 3 $^+$

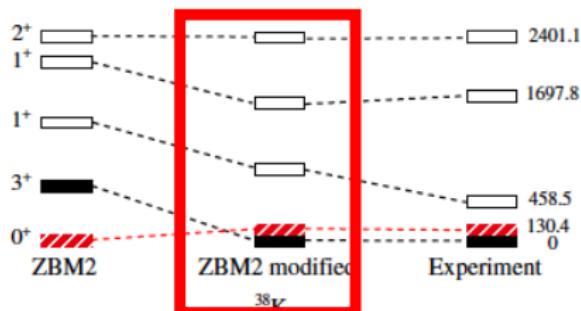


a correction of the ZBM2 is necessary preserving the description of the Ca isotopes shift

SM RESULTS : MODIFIED ZBM2

$$\mathcal{H}_m = \sum \varepsilon_i n_i + \sum a_{ij} n_i \cdot n_j + b_{ij} (T_i \cdot T_j - \frac{3}{4} n \delta_{ij})$$

modified ZBM2 : monopole correction of b_{ij} to $(d_{3/2})^2$ with $\Delta a_{jj} = 0$



✓ good level scheme

	$n_{fp}^\pi(38m)$	$n_{fp}^\pi(38g)$	$\delta\langle r_c^2 \rangle^{38g,38m} (\text{fm}^2)$
ZBM2	0.86	0.50	0.075
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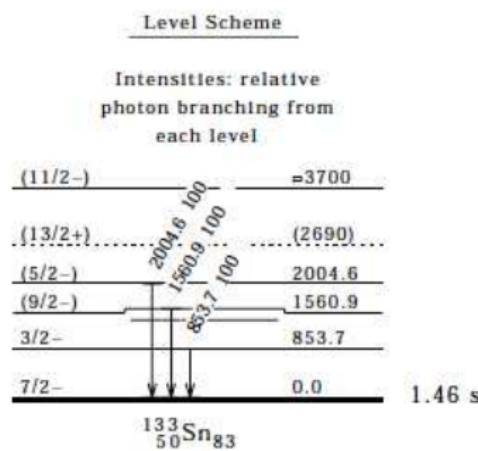
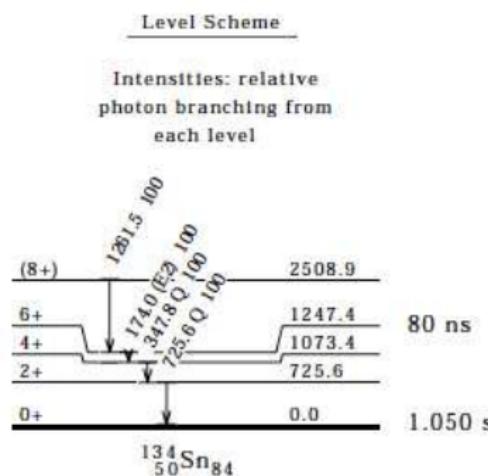
✓ good variation of r_c

The modified version of ZBM2 give us a correct order of the states, with conserving a similar composition of the wave functions.

PART II : SPECTROSCOPIC PROPERTIES OF THE NUCLEI AROUND ^{132}Sn

Introduction

Introduction

Adopted Levels, GammasAdopted Levels, Gammas ^{133}Sn ^{134}Sn ^{135}Sn

BE/A keV

8310.091 (18)

8275.160 (24)

8230.687 (23)

Experimental interest

✉ New results of $^{136,138}Sn$ obtained at RIKEN Nishina center.

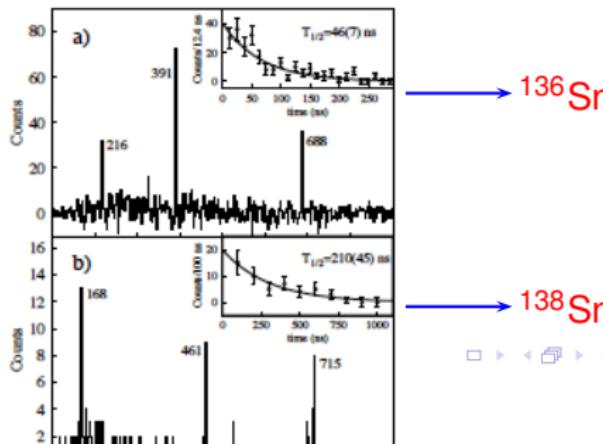
PRL 113, 132502 (2014)

PHYSICAL REVIEW LETTERS

week ending
26 SEPTEMBER 2014

Yrast 6^+ Seniority Isomers of $^{136,138}Sn$

G. S. Simpson,^{1,2,3} G. Gey,^{3,4,5} A. Jungclaus,⁶ J. Taprogge,^{6,7,5} S. Nishimura,⁵ K. Sieja,⁸ P. Doornenbal,⁵ G. Lorusso,⁵ P.-A. Söderström,⁵ T. Sumikama,⁹ Z. Y. Xu,¹⁰ H. Baba,⁵ F. Browne,^{11,5} N. Fukuda,⁵ N. Inabe,⁵ T. Isobe,⁵ H. S. Jung,^{12,*} D. Kameda,⁵ G. D. Kim,¹³ Y.-K. Kim,^{13,14} I. Kojouharov,¹⁵ T. Kubo,⁵ N. Kurz,¹⁵ Y. K. Kwon,¹³ Z. Li,¹⁶ H. Sakurai,⁵ H. Schaffner,¹⁵ Y. Shimizu,⁵ H. Suzuki,⁵ H. Takeda,⁵ Z. Vajta,^{17,5} H. Watanabe,⁵ J. Wu,^{16,5} A. Yagi,¹⁸ K. Yoshinaga,¹⁹ S. Bönig,²⁰ J.-M. Daugas,²¹ F. Drouet,³ R. Gernhäuser,²² S. Ilieva,²⁰ T. Kröll,²⁰ A. Montaner-Pizá,²³ K. Moschner,²⁴ D. Mücher,²² H. Naidžia,^{8,15,25} H. Nishibata,¹⁸ F. Nowacki,⁸ A. Odahara,¹⁸ R. Orlandi,^{26,†} K. Steiger,²² and A. Wendt²⁴



Theoretical interest

G, ^{132}Sn core, $\pi(gdsh) \otimes v(hfpi)$

JOURNAL REVIEW C 76, 024313 (2007)

Effective interactions and shell model studies of heavy tin isotopes

M. P. Kartamyshev, T. Engeland, M. Hjorth-Jensen, and E. Osnes

Department of Physics and Centre of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway

PHYSICAL REVIEW C 81, 064328 (2010)

New shell closure for neutron-rich Sn isotopes

SMPN, ^{132}Sn core, $\pi(gdsh) \otimes v(hfpi)$

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Indian Association for the Cultivation of Science University, Shibpur, Howrah 711103, India

M. Saha Sarkar†

Nuclear Physics Division, Saha Institute of Nuclear Physics, Kolkata 700064, India

Shell-model study of exotic Sn isotopes with a realistic effective interaction

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¹Dipartimento di Scienze Fisiche, Università di Napoli Federico II,

Viale S. Angelo, I-80126 Napoli, Italy

²Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy

V_{low-k} , ^{132}Sn core, $\pi(gdsh) \otimes v(hfpi)$

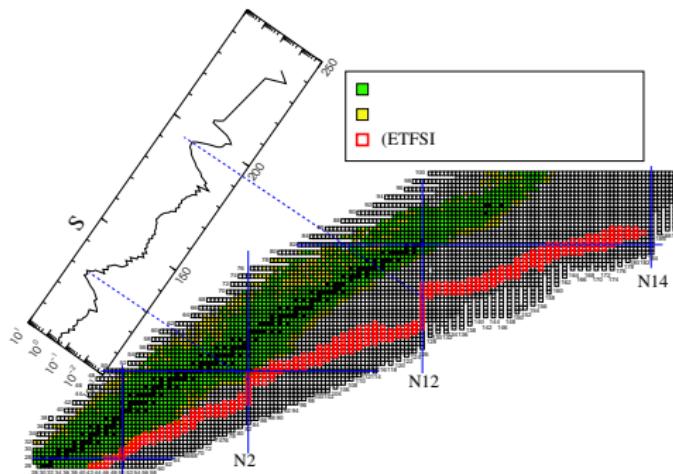
Viale S. Angelo, I-80126 Napoli, Italy



Interest field : Astrophysical

Astrophysical interest

- ☞ responsible of the synthesis of the heavy elements by r-process, and their nuclear model properties predictions give the inputs for r-process simulations.



Adapted from K.-L.Kratz

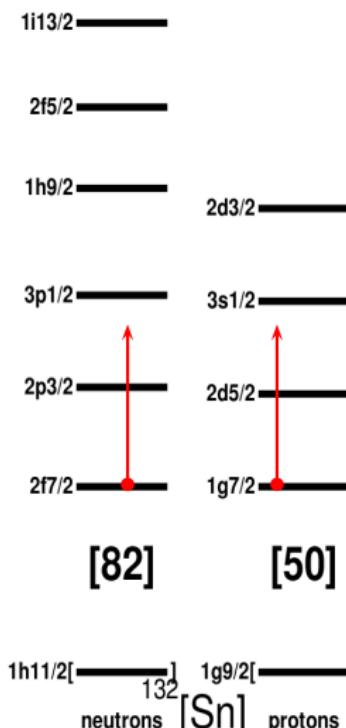
Core and valence space

Core and valence space



Core and valence space

Core and valence space



- ☒ 1h_{11/2} and 1g_{9/2} closed $\equiv ^{132}Sn$ core
- ☒ 1h_{11/2} and 1g_{9/2} opened $\equiv ^{110}Zr$ core
- ✓ Opening the ^{132}Sn core constitutes a numerical challenge in the diagonalisation of the matrix.

↓
 Diagonalization in **Antoine*** and **Nathan**† codes using Lanczos procedure,
 Exemple : ^{140}Sm : D=10 10¹⁰

(*).E.Caurier et al, Rev.Mod.Phys 77 (2007) 427, and Antoine website [http://antoine.cea.fr/](#)
 (†)no public version

EFFECTIVE INTERACTION

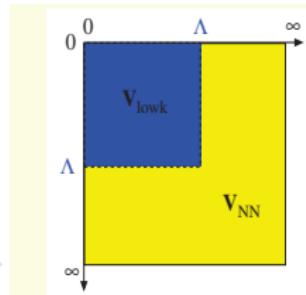
REALISTIC

1. derived from realistic interaction :ArgonneV18, CD-Bonn,N3LO,...
2. renormalised by V_{low-k} or G matrix approach to exclude the repulsive part at short range.
3. adapted to the model space by many body perturbation theory, using \hat{P} and \hat{Q} projection operators into **model space** and **excluded space** respectively

$$P = \sum_{i=1}^d |\Psi_i\rangle\langle\Psi_i|, Q = \sum_{i=d}^{\infty} |\Psi_i\rangle\langle\Psi_i|, P+Q=1$$

$$V_{eff} = V + \underbrace{V \frac{Q}{E - H_0} V}_{second\ order} + V \frac{Q}{E - H_0} V \frac{Q}{E - H_0} V$$

$$V \rightarrow V_{low-k}$$



Single particle energies

$13/2^+$ ————— 2691 $13/2^+$ ————— 2694

$1/2^+$ ————— 2800 $1/2^+$ ————— 2804

$3/2^+$ ————— 2439 $3/2^+$ ————— 2443

$5/2^-$ ————— 2005 $5/2^-$ ————— 2005

$9/2^-$ ————— 1561 $9/2^-$ ————— 1562

$1/2^-$ ————— 1363 $1/2^-$ ————— 1362

$3/2^-$ ————— 854 $3/2^-$ ————— 853

$5/2^+$ ————— 962 $5/2^+$ ————— 966

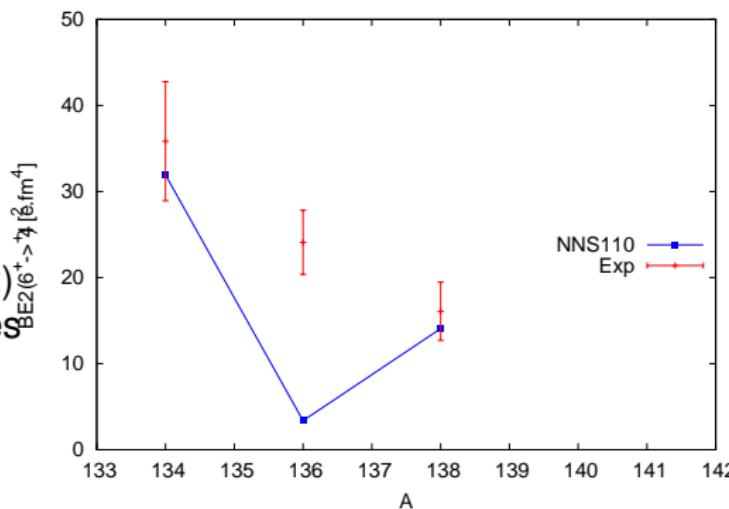
$7/2^-$ ————— 0 Exp. ^{133}Sn SM 0

$7/2^+$ ————— 0 Exp. ^{133}Sb SM 0

The re-adjustments of the monopole part of V_{low-k} interaction to obtain the experimental level scheme of ^{133}Sn and ^{133}Sb and their masses relative to ^{132}Sn .

B(E2, $6^+ \rightarrow 4^+$)

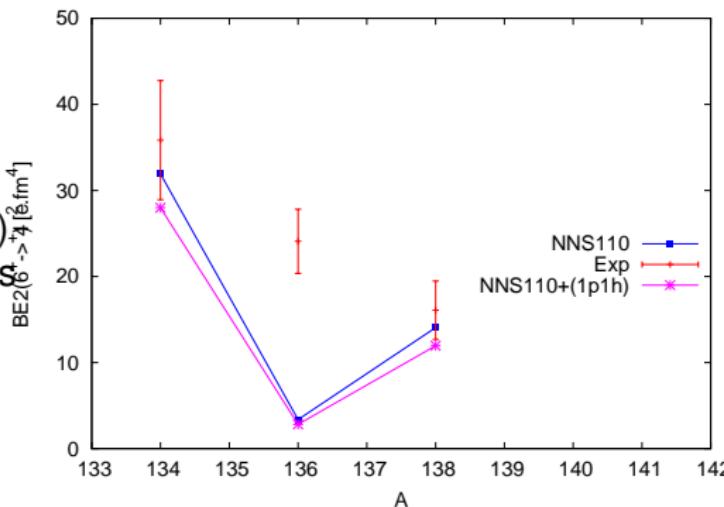
- ✗ **NNS110** : predicts well the B(E2) $_{BE2(6^+ \rightarrow 4^+)}^+$ in $^{134,138}Sn$, but it underestimates it for ^{136}Sn .



$$e_{eff}(v) = 0.64e$$

B(E2, $6^+ \rightarrow 4^+$)

- ✗ **NNS110** : predicts well the B(E2) in $^{134,138}Sn$, but it underestimates it for ^{136}Sn .
- ✗ **open core** : still underestimated of ^{136}Sn .



$$e_{eff}(\nu) = 0.5e$$

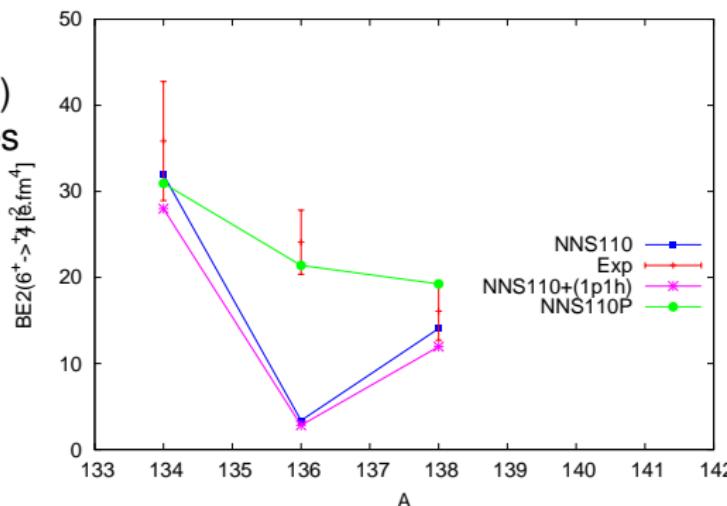
$$e_{eff}(\pi) = 1.5e$$

B(E2, $6^+ \rightarrow 4^+$)

✗ **NNS110** : predicts well the B(E2) in $^{134,138}Sn$, but it underestimates it for ^{136}Sn .

✗ **open core** : still underestimated of ^{136}Sn .

✓ **reducing the pairing (NNS110P)** : gives us a good agreement with the Exp.



H.Naidja et al. J.Phys.Conf.series, 580 (2015)012030

seniority

seniority mixing

$v=4$ 84%



$v=2$ 96%



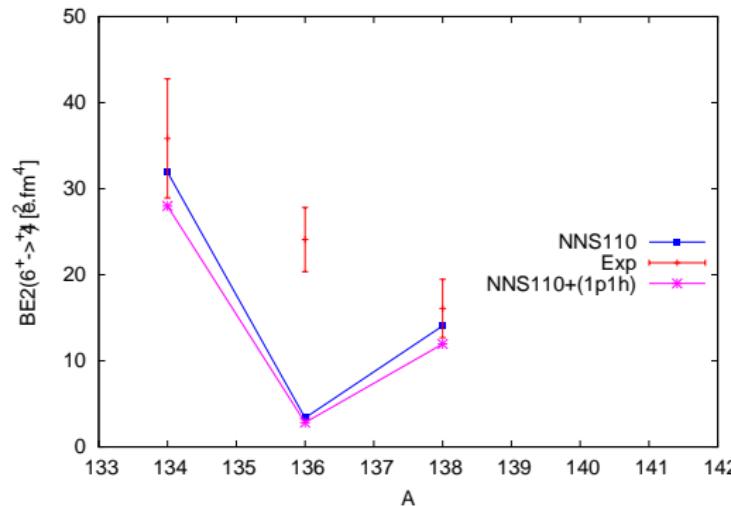
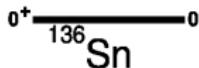
$v=2$ 82%



$v=2$ 95%



$v=0$ 98%



$$B(E2, v_f = v_i, J \rightarrow J-2) \propto (1 - \frac{2n}{2j+1})^2$$

- ▶ 6^+ and 4_1^+ are dominantly seniority $v = 2 \Rightarrow$ vanishing $B(E2)$
- ▶ $4_2^+(v = 4)$ is above the 6^+

seniority

seniority mixing

$\nu=2$ 95%

6^+ ————— 1221

4^+ ————— 1135

$\nu=2$ 45%, $\nu=4$ 55%

4^+ ————— 981

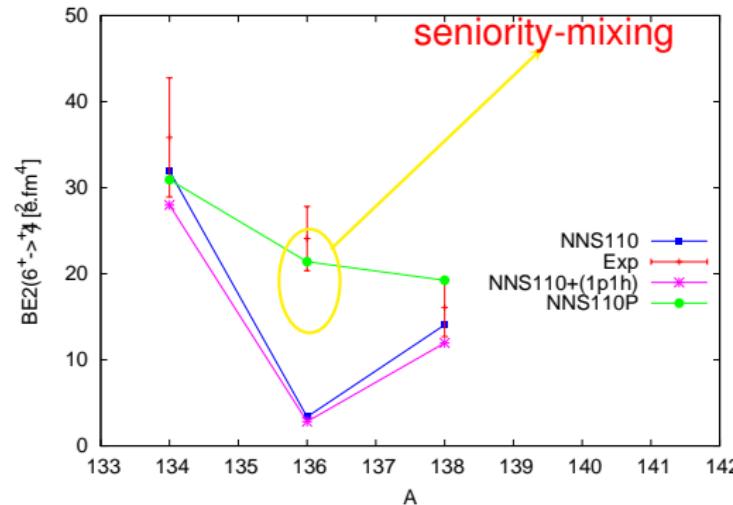
$\nu=2$ 52%, $\nu=4$ 48%

$\nu=2$ 93%

2^+ ————— 633

$\nu=0$ 98%

0^+ ————— ^{136}Sn



$$B(E2, \nu_f = \nu_i, J \rightarrow \nu_i, J-2) \propto (1 - \frac{2n}{2j+1})^2$$

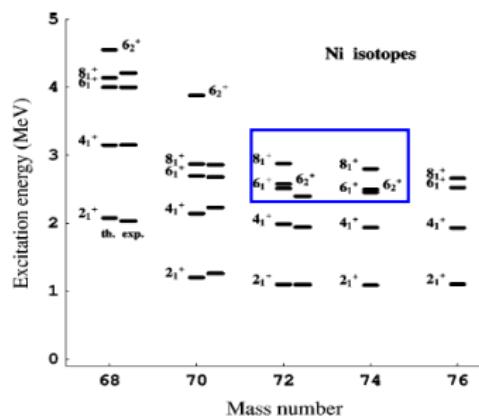
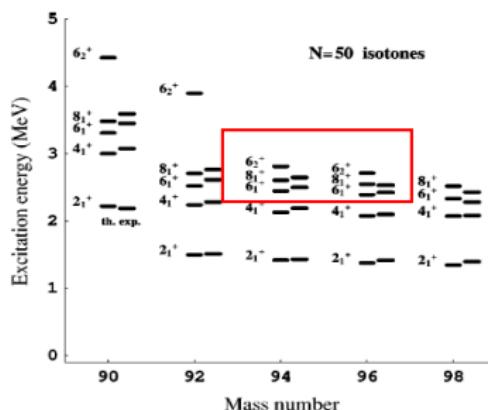
- ▶ 4_2^+ is below the 6^+
- ▶ mixed seniority $\nu = 2$ and $\nu = 4$ in 4_1^+ and 4_2^+ states \Rightarrow allowed E2 transitions.

seniority mixing

PHYSICAL REVIEW C 70, 044314 (2004)

New $T=1$ effective interactions for the $f_{5/2}$ $p_{3/2}$ $p_{1/2}$ $g_{9/2}$ model space:
Implications for valence-mirror symmetry and seniority isomers

A. F. Lisetskiy,¹ B. A. Brown,¹ M. Horoi,² and H. Grawe³



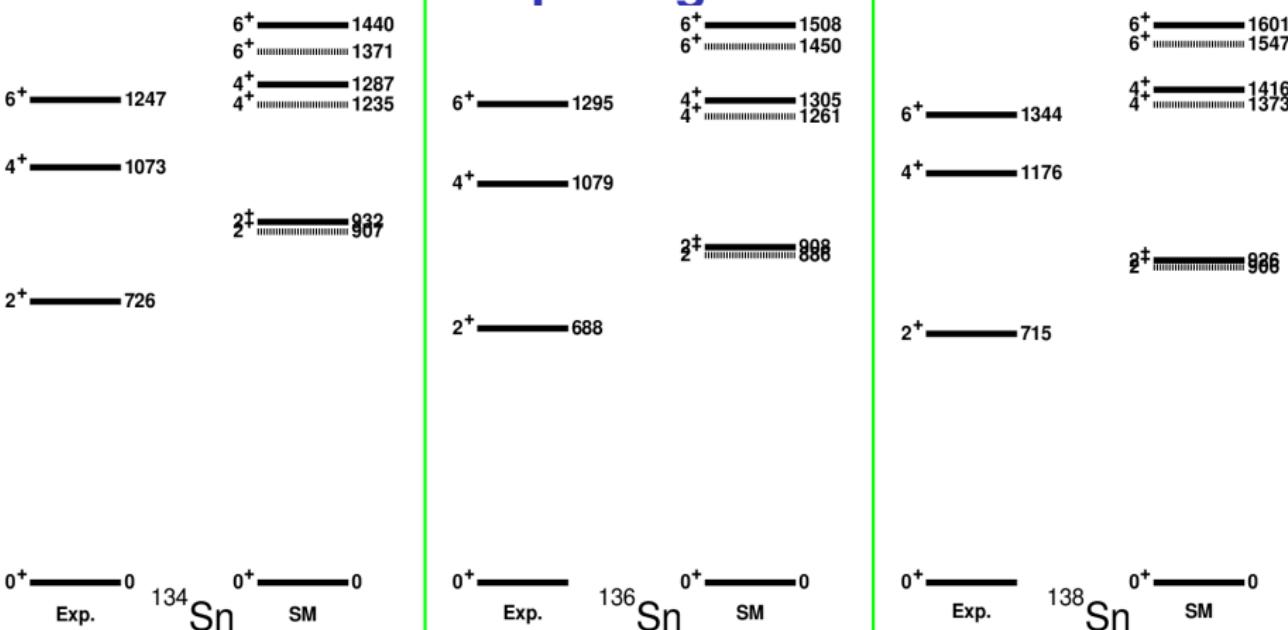
$6_2^+(\nu = 4)$ is above the 8^+

$6_2^+(\nu = 2 \text{ and } 4)$ is below the 8^+

Pushing down of the $6_2^+(\nu = 4)$ state opens up a new channel for the fast E2 decay of the 8^+ state.

tins : core

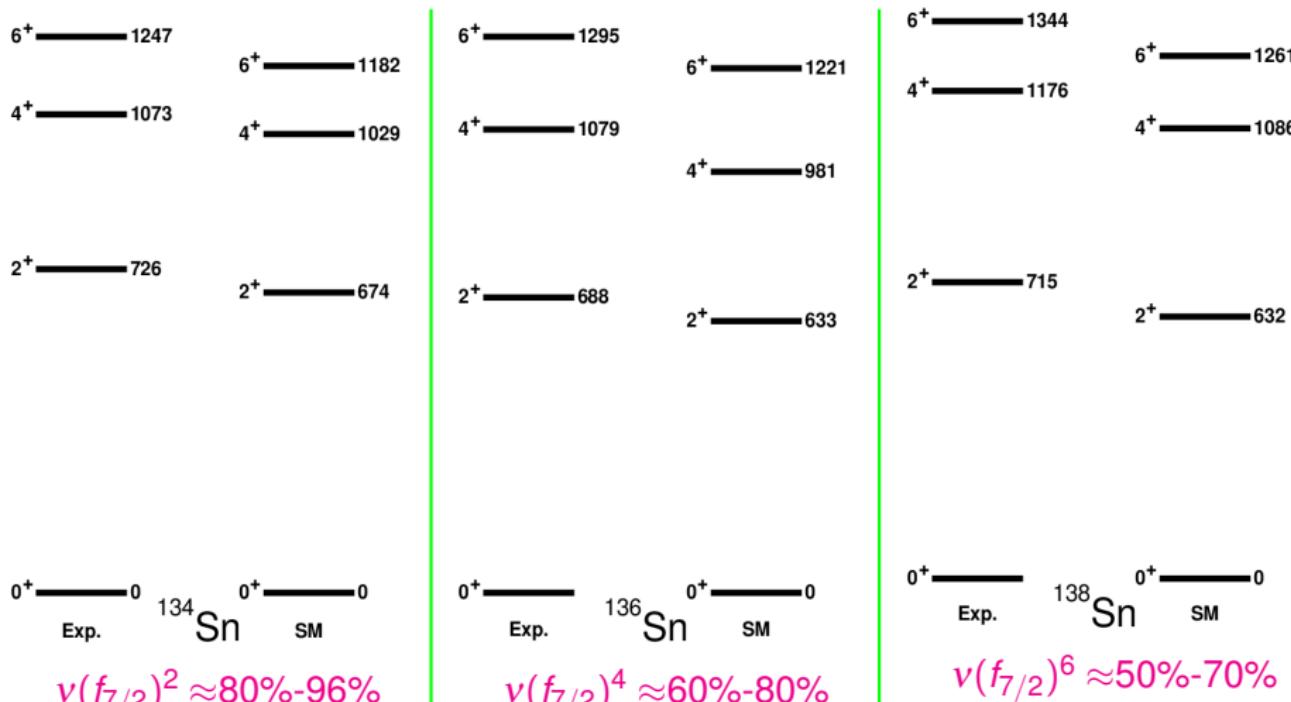
Tins : NNS110 and opening the core



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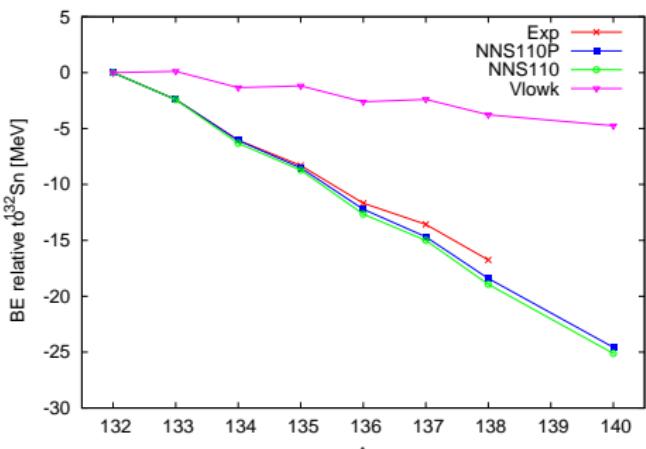
Small effect of the core excitations (1p1h) to the level scheme of tin isotopes.

Tins : NNS110P interaction



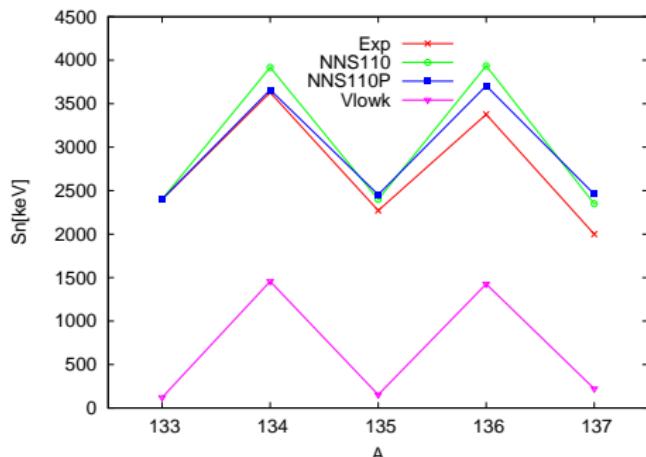
Masses

- ▶ Binding energies relative to ^{132}Sn



H.Naidja et al. J.Phys.Conf.series, 580 (2015)012030

- ▶ one neutron separation energy



- ✓ Our masses are consistent with the data
- ✓ The 2 body monopole corrections are believed to come from 3 body interaction not included in V_{low-k}

sub-shell closure at N=90 ?

PHYSICAL REVIEW C 81, 064328 (2010)

New shell closure for neutron-rich Sn isotopes

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(Received 11 October 2009; revised manuscript received 11 June 2010; published 29 June 2010)

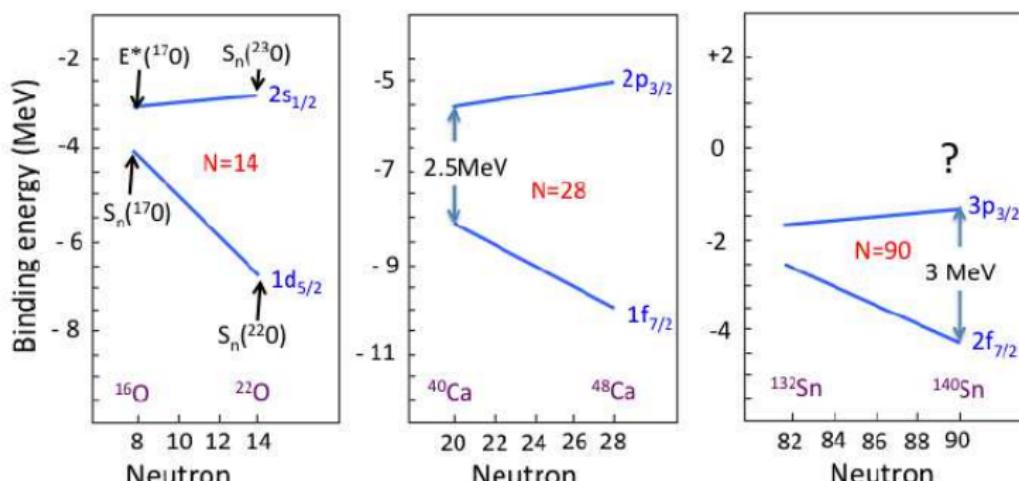
sub-shell closure at N=90 ?

PHYSICAL REVIEW C 81, 064328 (2010)

New shell closure for neutron-rich Sn isotopes

Department

Nuc
(Received



Taken from O.Sorlin notes

- Analogy between ^{22}O , ^{48}Ca , and ^{140}Sn in the closure of the (sub-)shell at N=14,28, and 90?

A.P.Zuker, PRL 90, 042502 (2003)

T.Otsuka et al. PRL 105, 032501 (2010)

Sub-shell closure at N=90 ?

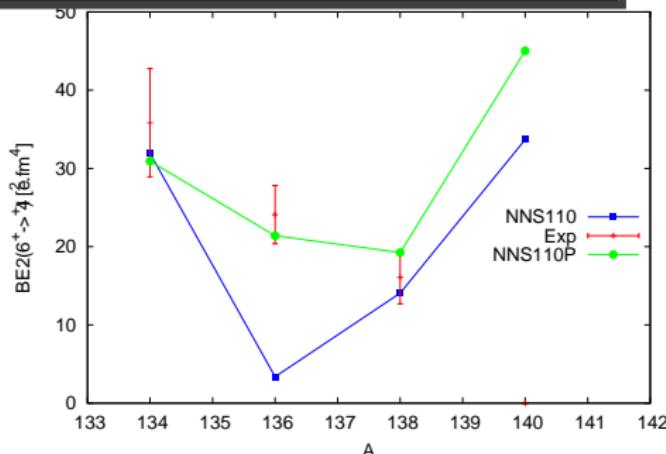
$v(f_{7/2})^6(p_{3/2})^2$ 6^+ ————— 1661

$v(f_{7/2})^7(p_{3/2})^1$ 4^+ ————— 1178

$v(f_{7/2})^6(p_{3/2})^2$ 2^+ ————— 780

$v(f_{7/2})^8$ 0^+ ————— 0
 ^{140}Sn

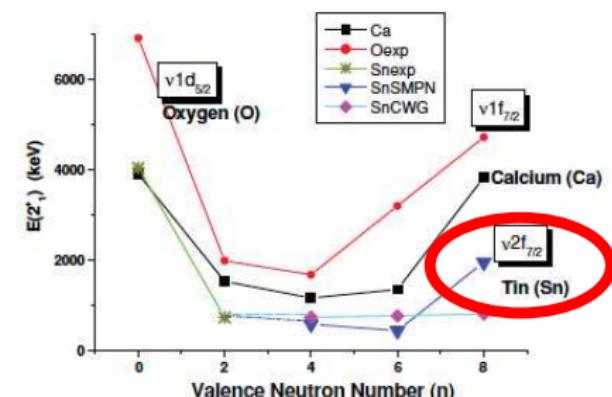
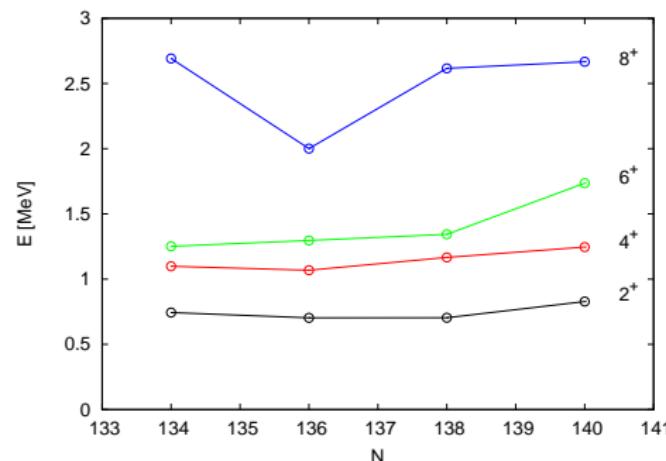
our predictions to ^{140}Sn structure using **NNS110P**



H.Naïdja and al. J.Phys.Conf series, 580 (2015)012030

- ▶ the excited states are characterized by mixed configurations.

Sub-shell closure at N=90 ?



H.Naïdja et al. J.Phys.Conf.series, 580 (2015)012030

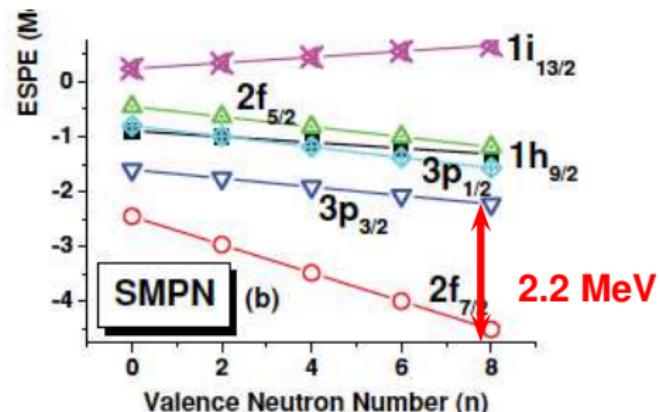
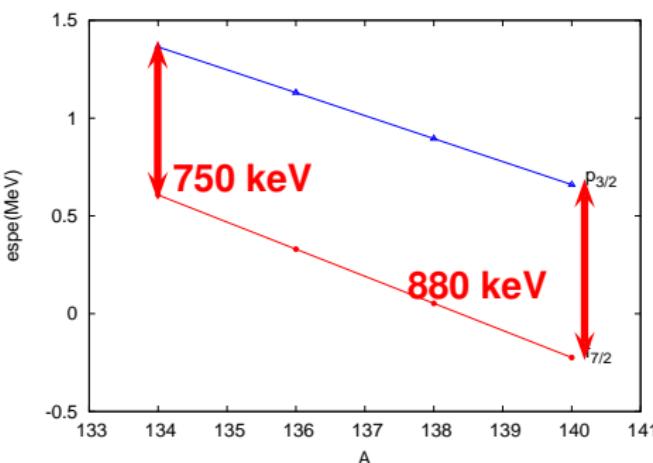
- the spacing $0^+ - 2^+$ remains nearly constant at around 700 keV, except for a small increase at ^{140}Sn owing to the filling of the $(f_{7/2})$.

2^+ energy in ^{140}Sn is not higher than in the neighboring nuclei.

S.Sarkar and M.S.Sarkar Phys.Rev.C 81, 064328(2010)

- a sudden increase for N=90, indicating a closed-shell structure for ^{140}Sn .

Evolution of neutron ESPE



H.Naïdja et al. J.Phys.Conf.series, 580 (2015)012030

S.Sarkar and M.S.Sarkar Phys.Rev.C 81, 064328(2010)

- The gap between $vf_{7/2}$ and $vp_{3/2}$ remains constant



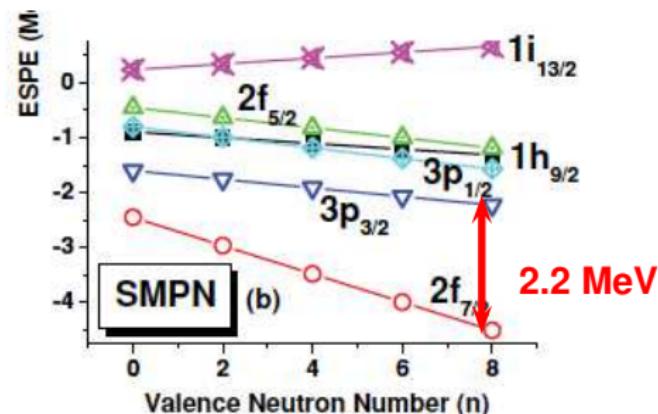
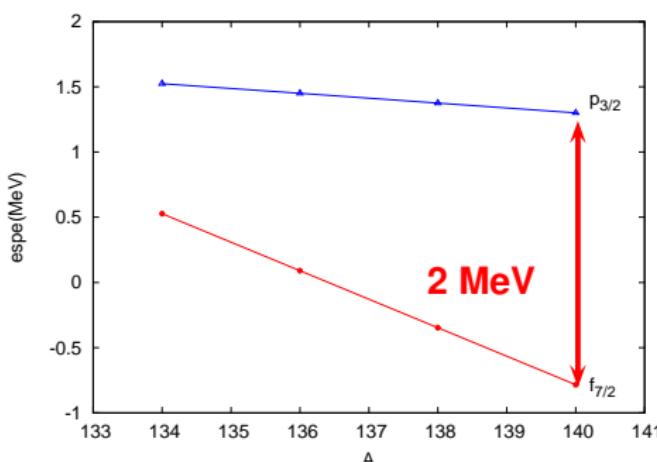
No sub-shell closure at N=90

- The gap $vf_{7/2}$ and $vp_{3/2}$ increases to 2.246 MeV



^{140}Sn is doubly magic nucleus

Evolution of neutron ESPE



S.Sarkar and M.S.Sarkar Phys.Rev.C 81, 064328(2010)

- Increasing the gap by changing the monopole part

- The gap $v f_{7/2}$ and $v p_{3/2}$ increases to 2.246 MeV



^{140}Sn is a doubly magic nucleus

Increasing the gap

$v(f_{7/2})^7(f_{5/2})^1$ $^{6+}$ $\xrightarrow{2628}$

$v(f_{7/2})^7(p_{3/2})^1$ $^{4+}$ $\xrightarrow{1905}$

$v(f_{7/2})^7(p_{3/2})^1$ $^{2+}$ $\xrightarrow{1810}$

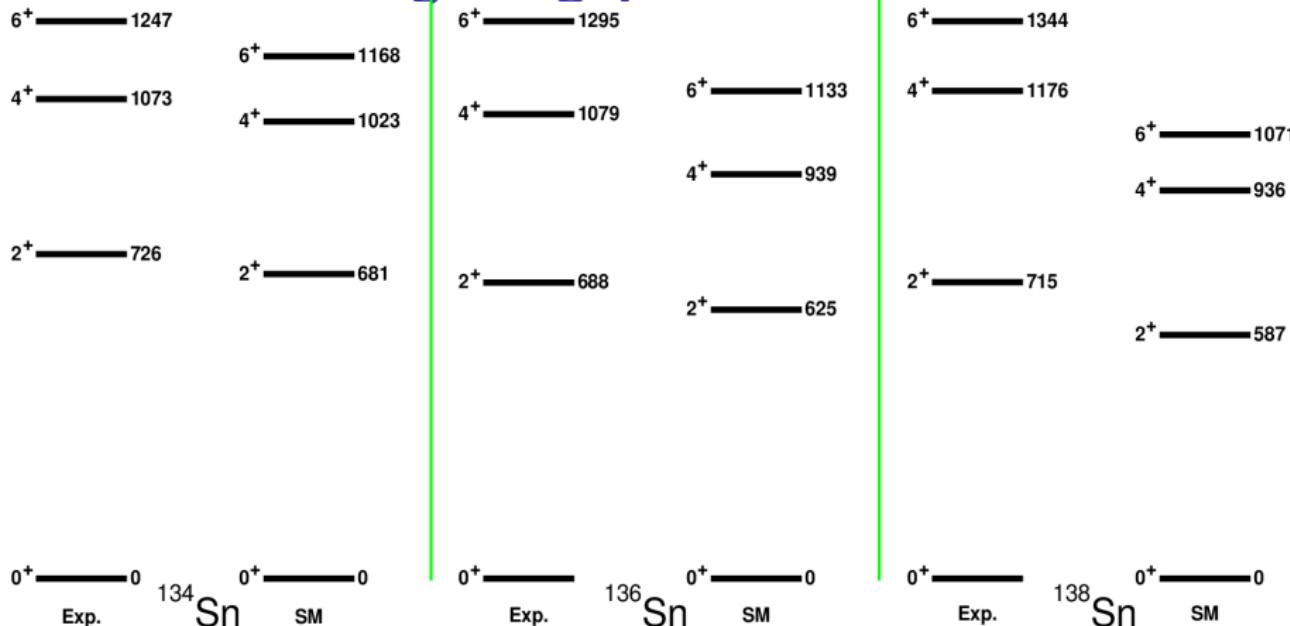
$v(f_{7/2})^8$ $^0+$ $\xrightarrow{140}$ ^{140}Sn

Increasing the gap by changing the monopole part



- ▶ Transitions are slightly inconsistent with the experience
- ▶ sudden decrease of $B(E2)$ in ^{140}Sn
 1. E(2) favor the transitions from $j \rightarrow j+2 (p_{3/2} \rightarrow f_{7/2})$ compared as $j \rightarrow j+1 (f_{5/2} \rightarrow f_{7/2})$

Tins : increasing the gap



increasing $f_{7/2} - p_{3/2}$ gap, has an apparent effect to ^{138}Sn structure.

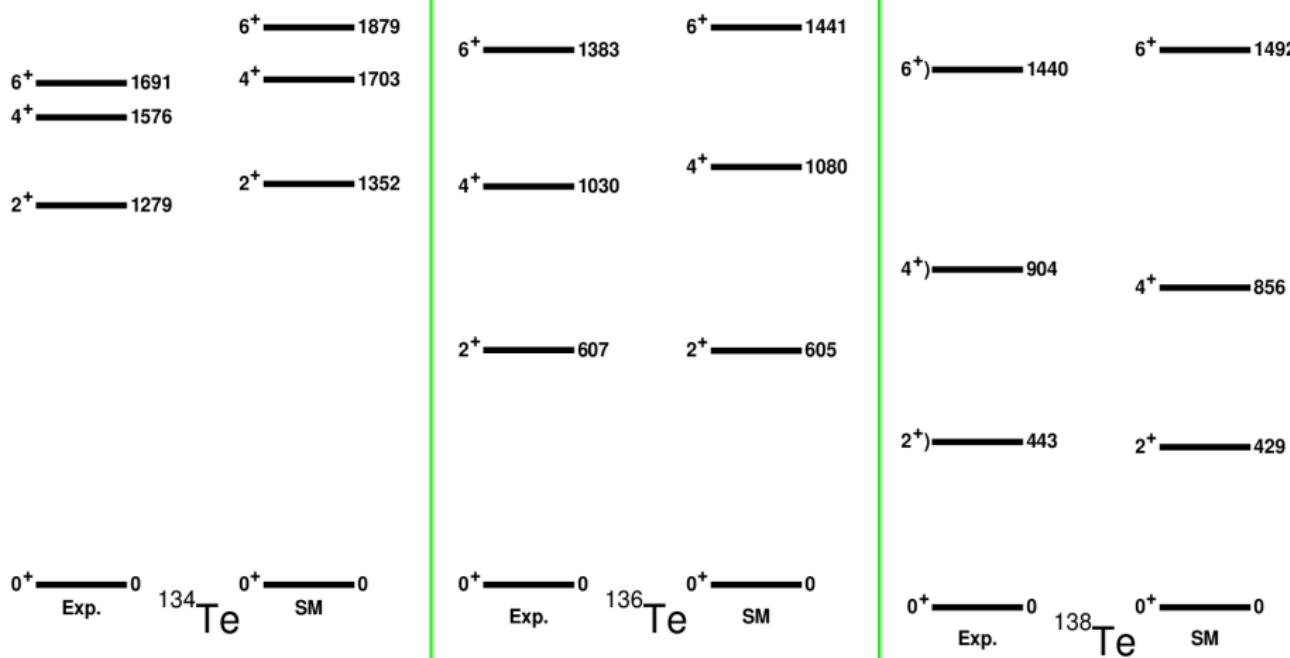


Losing the spectroscopic properties.

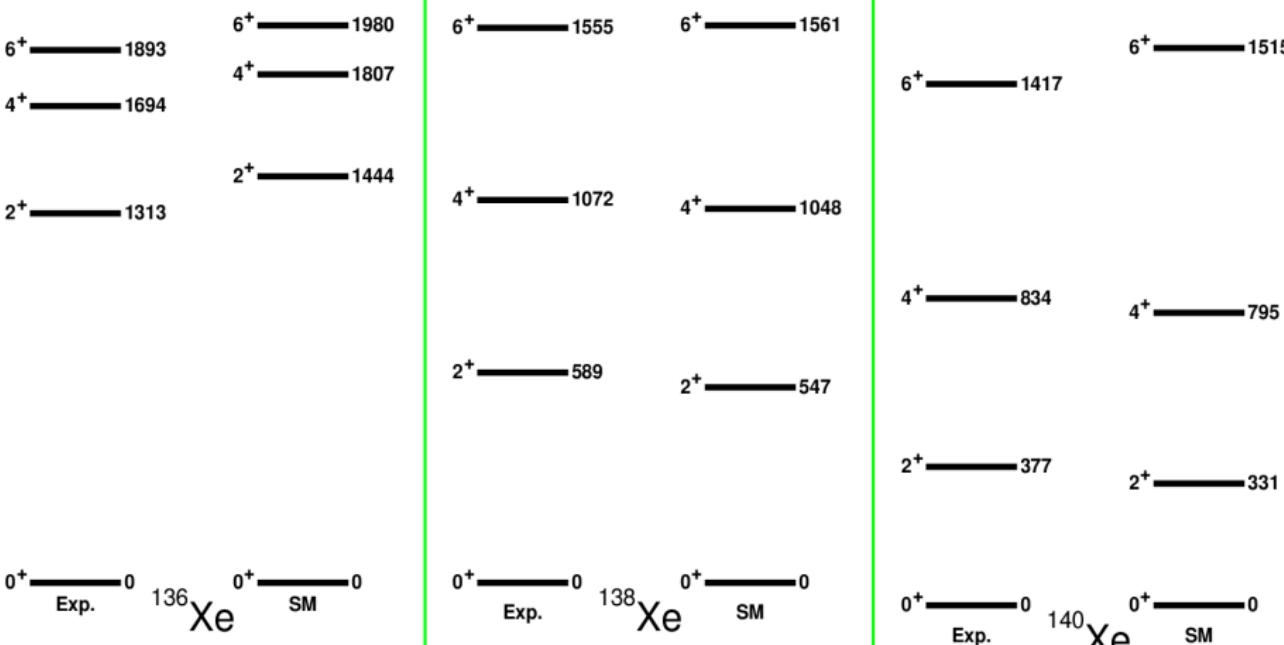
No sub-shell closure at N=90

OTHER APPLICATIONS TO THE OPEN N-P SYSTEMS

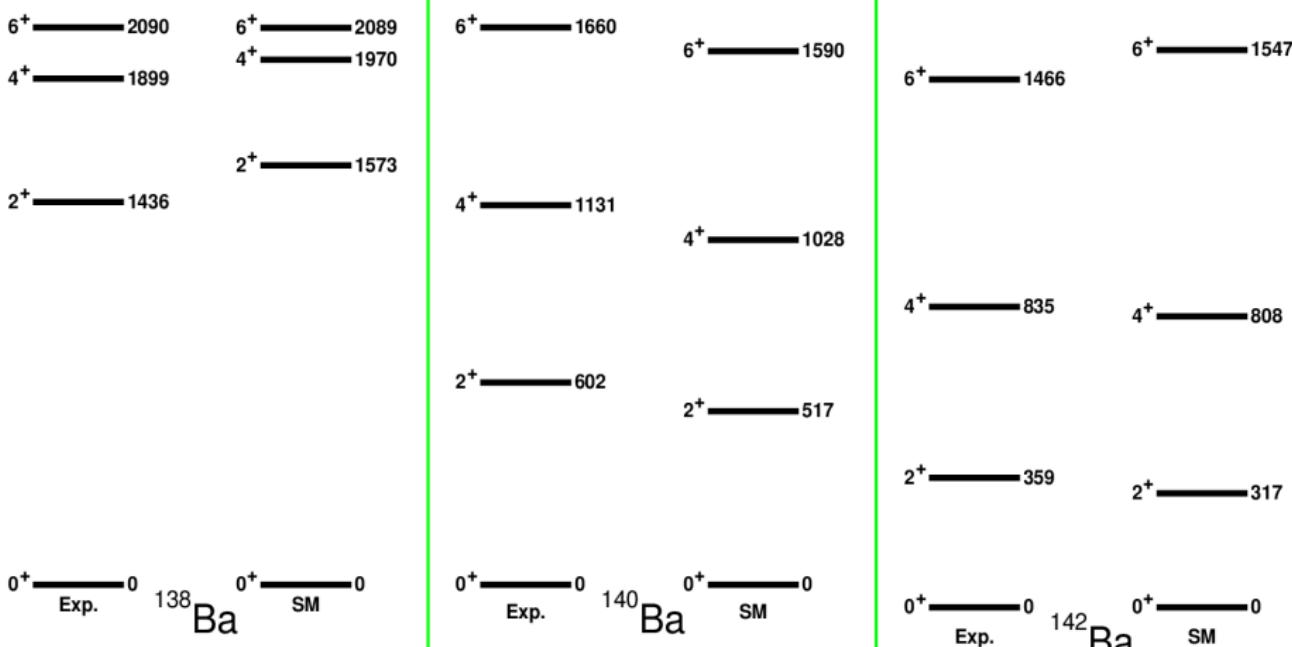
Tellerium



Xenon



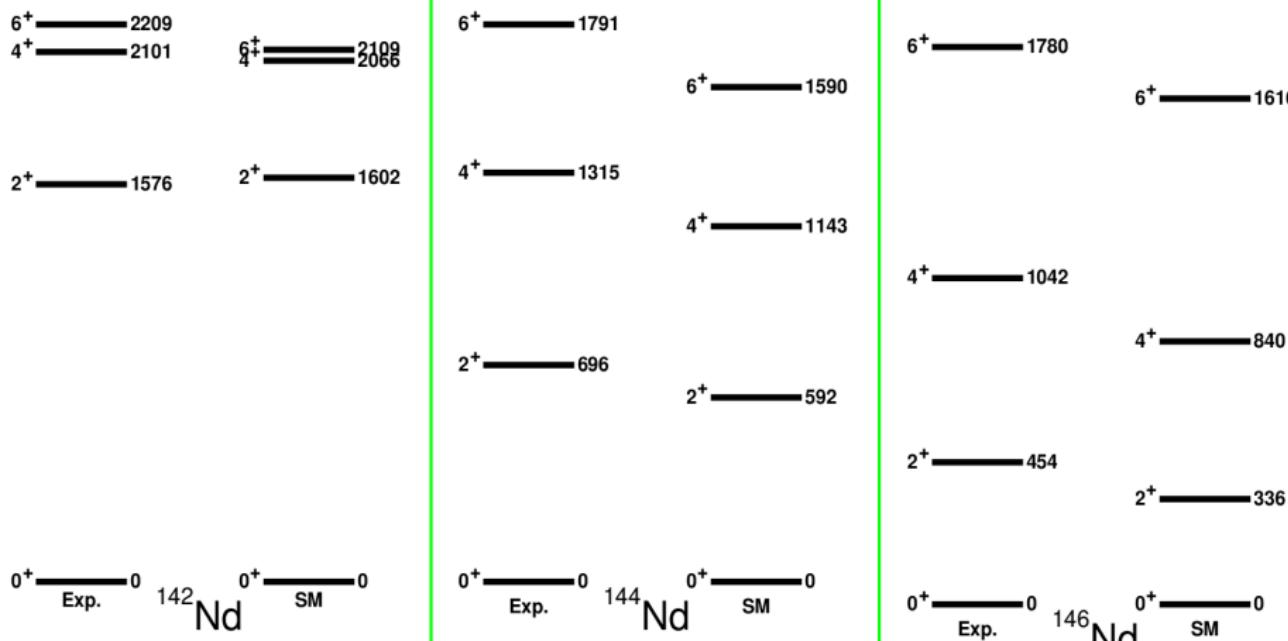
Barium



Cerium

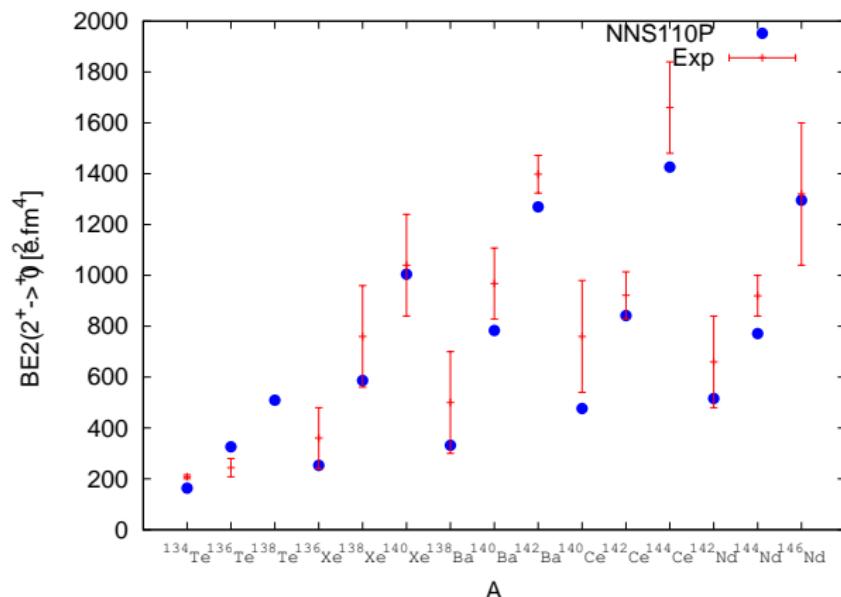
 6^+_4 ————— 2108
————— 2083 6^+_4 ————— 2082
————— 2076 2^+_2 ————— 1596 2^+_2 ————— 1629 6^+_6 ————— 1743 6^+_6 ————— 1610 6^+_6 ————— 1647 4^+_4 ————— 1219 4^+_4 ————— 1086 6^+_6 ————— 1516 2^+_2 ————— 641 2^+_2 ————— 544 4^+_4 ————— 789 0^+_0 ————— 140 Ce 0^+_0 ————— SM 0 0^+_0 ————— 142 Ce 0^+_0 ————— SM 0 0^+_0 ————— 144 Ce 0^+_0 ————— SM 0

Neodymium



Collective properties

$B(E2, 2^+ \rightarrow 0^+)$



To be published

$$e_{eff}(\pi) = 1.5e$$

$$e_{eff}(\nu) = 0.5e$$

- ✓ B(E2) are reproduced with no explicit adjustment of the e_{eff} .
- ✓ BE2 in ^{136}Te is close to that ^{134}Te , so anomaly in ^{136}Te

Triaxiality

Triaxiality

- ▶ $Q(2_1^+) = -Q(2_{yrast}^+)$.
- ▶ the presence of $B(E2, 2_1^+ \rightarrow 2_2^+)$ transition
- ▶ $Q(3^+) = 0$.
- ▶ strong $B(E2, 3^+ \rightarrow 2_2^+)$ transition

work under progress with Bounseng Bounthong using the deformed HF with constraints

	^{138}Te	^{140}Xe	^{142}Ba	^{144}Ce	^{146}Nd
$Q(2_1^+) e.fm^2$	-45.67	-62.64	-70.74	-75.24	-61.76
$Q(2_2^+) e.fm^2$	40.08	63.84	69.18	75.49	-60.92
$Q(3^+) e.fm^2$	-0.34	-1.31	-0.62	-0.79	-0.10
$B(2_2^+ \rightarrow 2_1^+) e^2.fm^4$	43	155	180	210	294
$B(3^+ \rightarrow 2_2^+) e^2.fm^4$	745	1911	2054	2569	2153
$Q_i(Q_0)$	157	220	248	262	248
$Q_i(B(E2))$	160	225	252	268	255
β	0.1	0.14	0.15	0.15	0.14

CONCLUSIONS-PERSPECTIVES

- ✓ Using NNS110P interaction, the agreement between the experience and calculated energy levels is improved.
- ✓ The pairing force must be reduced to reproduce the experimental transition rates in ^{136}Sn , leading to mixing seniority.
- ✓ The core excitations seem to have a negligible effect to the tin isotopes energies, confirming the strong magicity of ^{132}Sn
- ✓ ^{140}Sn doesn't exhibit the features of a doubly magic nucleus.
- ✓ The applications to other nuclei allowed us to test our interaction to differents systems.

PROJECTS UNDER PROGRESS

- ▶ Triaxiality in ^{140}Sm : in collaboration with Andreas Görzen, University of Oslo.
- ▶ Isomer in ^{140}Sb : in collaboration with Radomira Lozeva ; IPHC Strasbourg
- ▶ High spin states in ^{138}Te and ^{140}Xe : in collaboration with W.Urban,
- ▶ The deformation in the nuclei beyond ^{132}Sn with B.Bounthong.
- ▶ ...