Nonlocal Dispersive Optical Model

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Table of contents

Hartree-Fock Potential

In general we want to solve a problem with Hamiltonian

$$H = T + V$$

The irreducible self energy can be written as:

$$\Sigma^*(y,x;E) = -i\int \frac{dE'}{2\pi}\sum_{x',y'}\langle yx'|V|xy'\rangle G(y',x';E') + Higher\ Orders(E)$$

The Hartree-Fock approximation means to eliminate “higher orders” which in general depend on energy.

Dispersion Relation

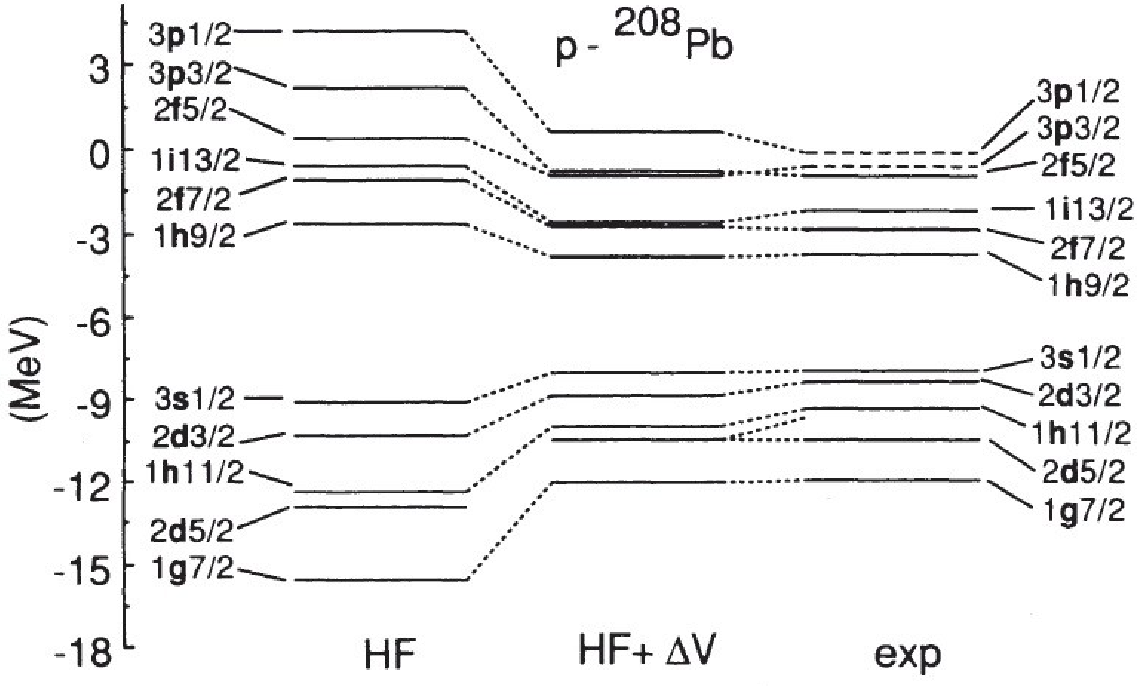
By evaluating the real part, let say at some energy ε_F, one can rewrite the dispersion relation as:

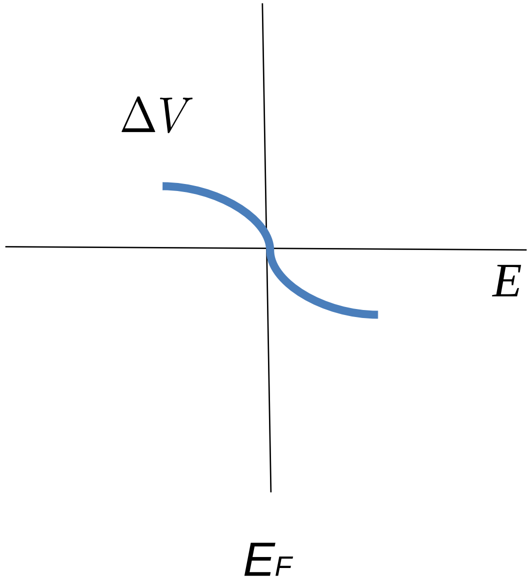
$$\mathrm{Re}\Sigma(x,y;E) = \Sigma_s(x,y) - \mathcal{P}\int_{\epsilon_T^+}^{\infty} \frac{dE'}{\pi}\frac{\mathrm{Im}\Sigma(x,y;E')}{E-E'} + \mathcal{P}\int_{-\infty}^{\epsilon_T^-} \frac{dE'}{\pi}\frac{\mathrm{Im}\Sigma(x,y;E')}{E-E'}$$

$$\mathrm{Re}\Sigma(x,y;\varepsilon_F) = \Sigma_s(x,y) - \mathcal{P}\int_{\epsilon_T^+}^{\infty} \frac{dE'}{\pi}\frac{\mathrm{Im}\Sigma(x,y;E')}{\varepsilon_F-E'} + \mathcal{P}\int_{-\infty}^{\epsilon_T^-} \frac{dE'}{\pi}\frac{\mathrm{Im}\Sigma(x,y;E')}{\varepsilon_F-E'}$$

$$\begin{aligned}\mathrm{Re}\Sigma(x,y;E) = \mathrm{Re}\Sigma(x,y;\varepsilon_F) &- \mathcal{P}\int_{\varepsilon_T^+}^{\infty} \frac{dE'}{\pi}\mathrm{Im}\Sigma(x,y;E') \times \left[\frac{1}{E-E'} - \frac{1}{\varepsilon_F-E'}\right] \\ &+ \mathcal{P}\int_{-\infty}^{\varepsilon_T^-} \frac{dE'}{\pi}\mathrm{Im}\Sigma(x,y;E') \times \left[\frac{1}{E-E'} - \frac{1}{\varepsilon_F-E'}\right]\end{aligned}$$

Effect of dispersion relation





C. Mahaux and R. Sartor, Adv. Nucl. Phys.

Dispersion Relation

Titchmarsh's Theorem

Establishes one-to-one correspondence between the existence of a Hilbert transform (***dispersion relation***), ***analyticity*** properties and ***causality***

$$L = \int dE \; F(E) \exp(-iE\tau)$$

vanishes for $\tau < 0$, where

$$F(F) = f(E) + i\, g(E)$$

C. Mahaux, K. T. R Davies and G. R. Satchler , Physics Reports 224. Nos.5 & 6 (1993)

Lehmann representation:

$$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\langle\Psi_0^N|\mathcal{T}[a_{\alpha_H}(t)a^\dagger_{\beta_H}(t')]|\Psi_0^N\rangle$$

$$G(\alpha,\beta;E) = \sum_m \frac{\langle\Psi_0^N|a_\alpha|\Psi_m^{N+1}\rangle\langle\Psi_m^{N+1}|a^\dagger_\beta|\Psi_0^N\rangle}{E-(E_m^{N+1}-E_0^N)+i\eta} + \sum_n \frac{\langle\Psi_0^N|a^\dagger_\beta|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|a_\alpha|\Psi_0^N\rangle}{E-(E_0^N-E_n^{N-1})-i\eta}$$

$$= \langle\Psi_0^N|a_\alpha\frac{1}{E-(\hat{H}-E_0^N)+i\eta}a^\dagger_\beta|\Psi_0^N\rangle + \langle\Psi_0^N|a^\dagger_\beta\frac{1}{E-(E_0^N-\hat{H})-i\eta}a_\alpha|\Psi_0^N\rangle$$

$$\boxed{G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G(\alpha,\gamma;E)\Sigma^*(\gamma,\delta;E)G^{(0)}(\delta,\beta;E)}$$

Dyson equation

Local DOM Potential

$$\mathcal{U} = \mathcal{V} + i\mathcal{W}$$

$$\mathcal{W}(r, E) = -\mathcal{W}_v(r, E) f(r, r_v, a_v) + 4 a_s W_s(E) \frac{d}{dr} f(r, r_s, a_s) + \mathcal{W}_{so}(r, E)$$

$$\mathcal{V}(r, E) = \mathcal{V}_{HF}(r, E) + \Delta\mathcal{V}(r, E)$$

$$\Delta\mathcal{V}(r, E) = \frac{1}{\pi}\mathcal{P}\int \mathcal{W}(r, , E')\left(\frac{1}{E' - E} - \frac{1}{E' - \varepsilon_F}\right) dE'$$

$$\mathcal{V}_{HF}(r, E) = -V_{HF}^{Vol}(E) f(r, r_{HF}, a_{HF}) + 4 V_{HF}^{Sur} \frac{d}{dr} f(r, r_{HF}, a_{HF}) + V_c(r) + \mathcal{V}_{so}(r, E)$$

Where

$$f(r, r_i, a_i) = \frac{1}{1 + e^{\frac{r - r_i A^{1/3}}{a_i}}}$$

PHYSICAL REVIEW C **82**, 054306 (2010)

Nonlocal extension of the dispersive optical model to describe data below the Fermi energy

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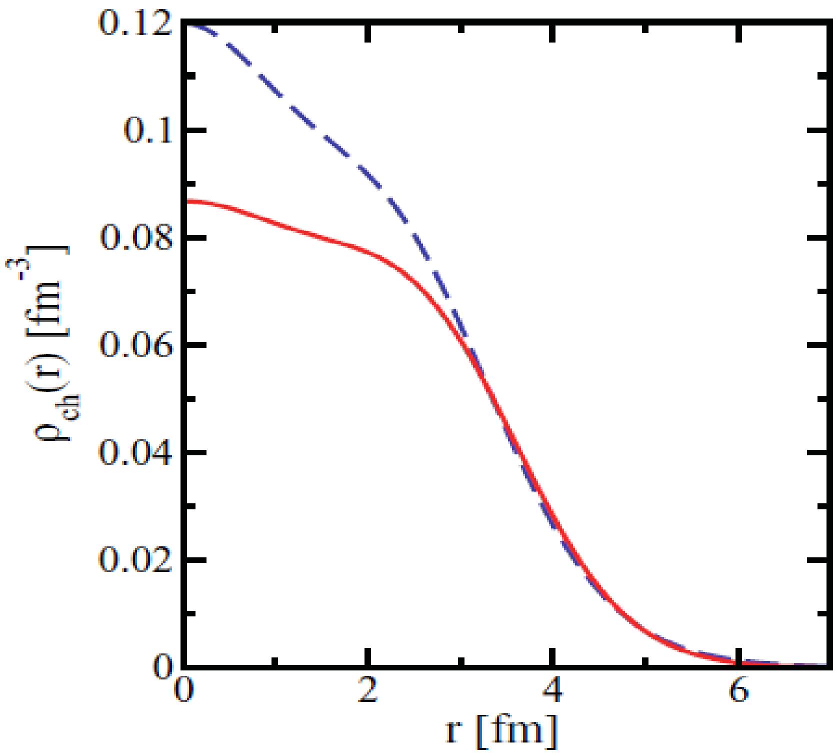
(Received 1 October 2010; published 10 November 2010)

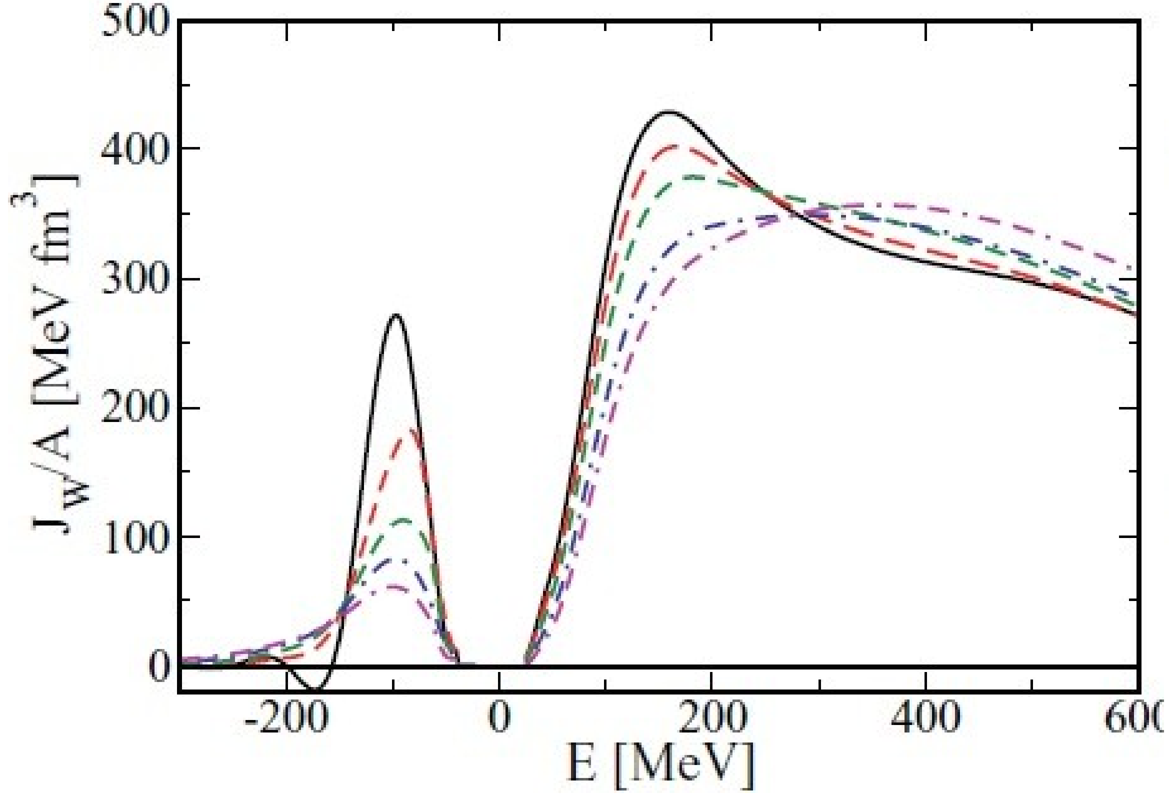
$$\sum_{HF}(r,r') = V_{NL} f(\frac{1}{2}|r+r'|)H(|r-r'|)$$

$$H(|\boldsymbol{r}-\boldsymbol{r}'|) = \frac{1}{\pi^{\frac{3}{2}}\beta^3}\exp\left[-\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\beta}\right)^2\right]$$



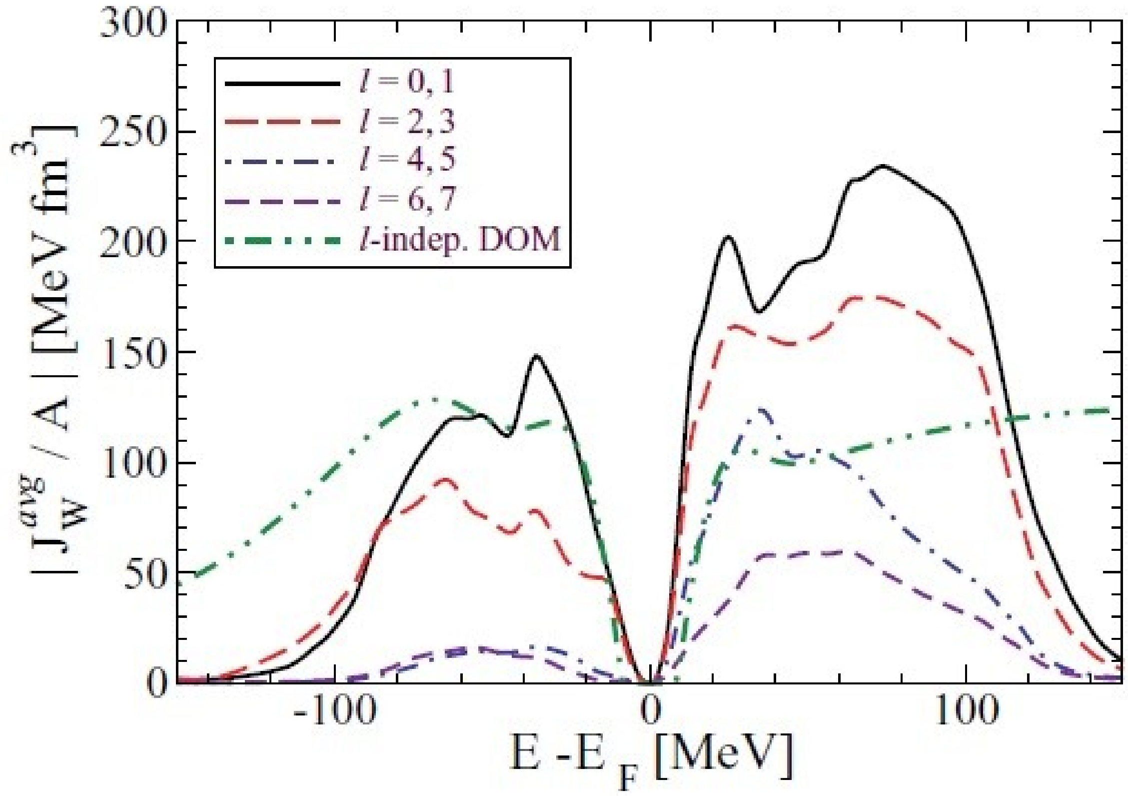
Role of Nonlocality

CDBonn (short range)



Phys. Rev. C 84, 044319 (2011)

FRPA



Phys. Rev. C 84, 034616 (2011)

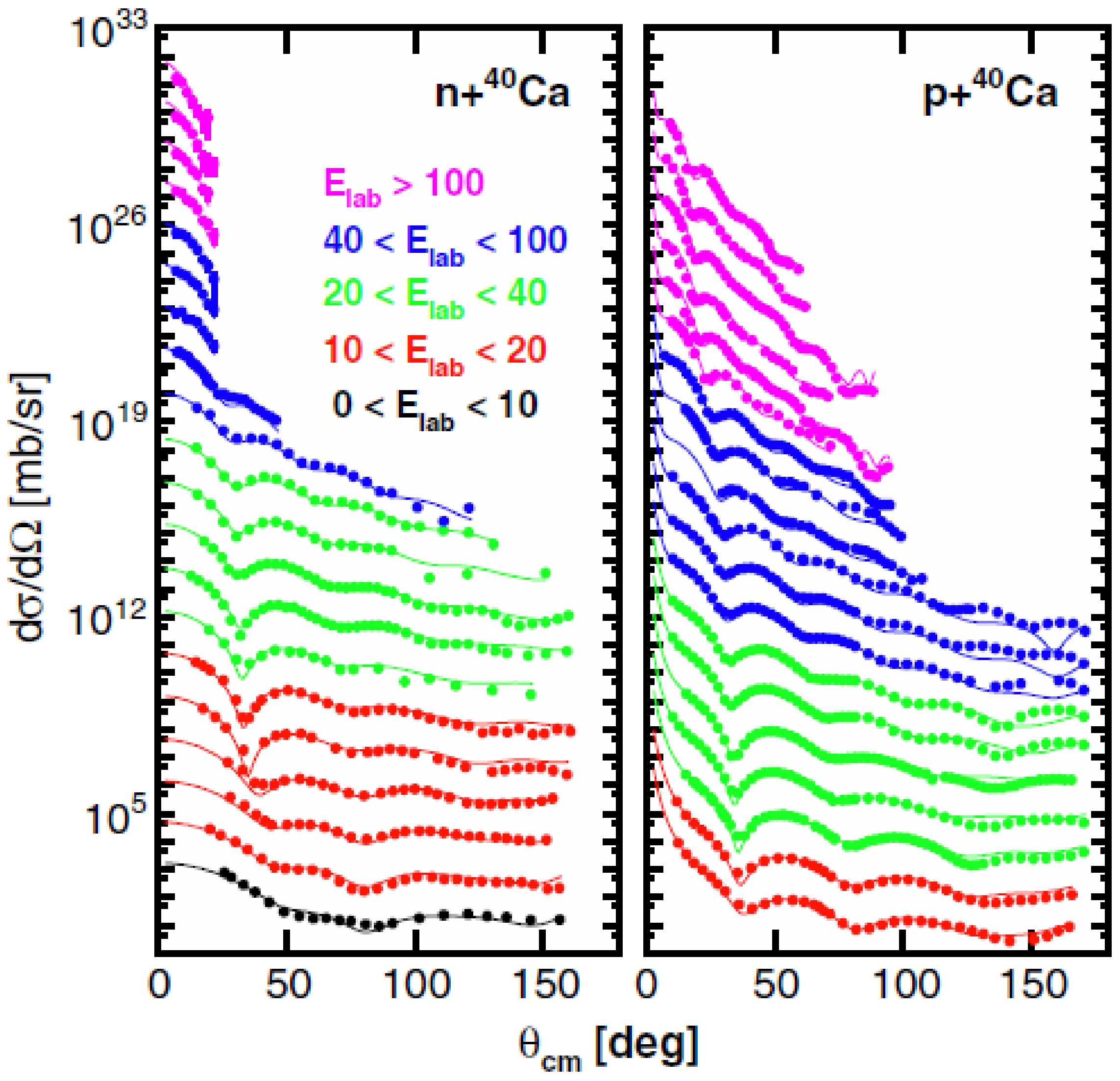
Nonlocal DOM

- For Example:

$$\mathrm{Im}\ \Sigma(\boldsymbol{r},\boldsymbol{r}',E) = \mathrm{Im}\ \Sigma^{nl}(\boldsymbol{r},\boldsymbol{r}';E) + \delta(\boldsymbol{r}-\boldsymbol{r}')\mathcal{W}^{so}(r;E)$$

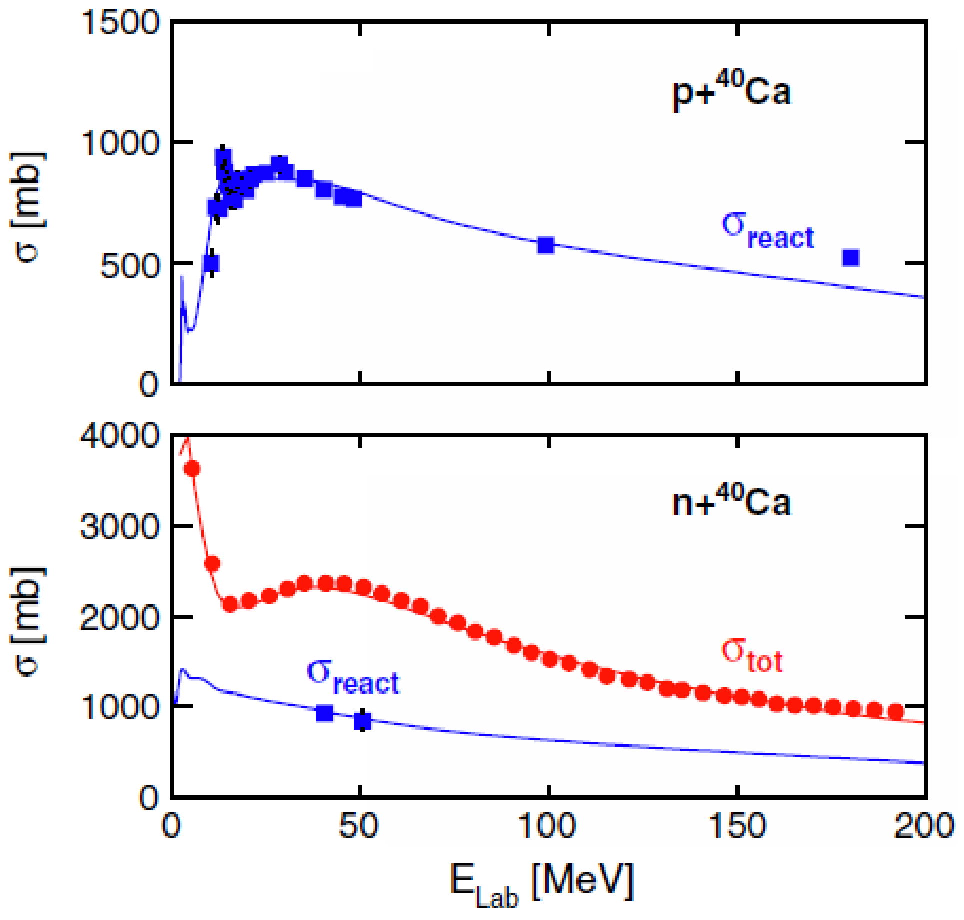
$$\mathrm{Im}\ \Sigma^{nl}(\boldsymbol{r},\boldsymbol{r}';E) = -W^{vol}_{0\pm}(E)f\left(\tilde{r};r^{vol}_{\pm};a^{vol}_{\pm}\right)H\left(\boldsymbol{s};\beta^{\pm}_{vol}\right)$$
$$+4a^{sur}_{\pm}W^{sur}_{\pm}(E)\,H\left(\boldsymbol{s};\beta^{\pm}_{sur}\right)\frac{d}{d\tilde{r}}f(\tilde{r},r^{sur}_{\pm},a^{sur})$$

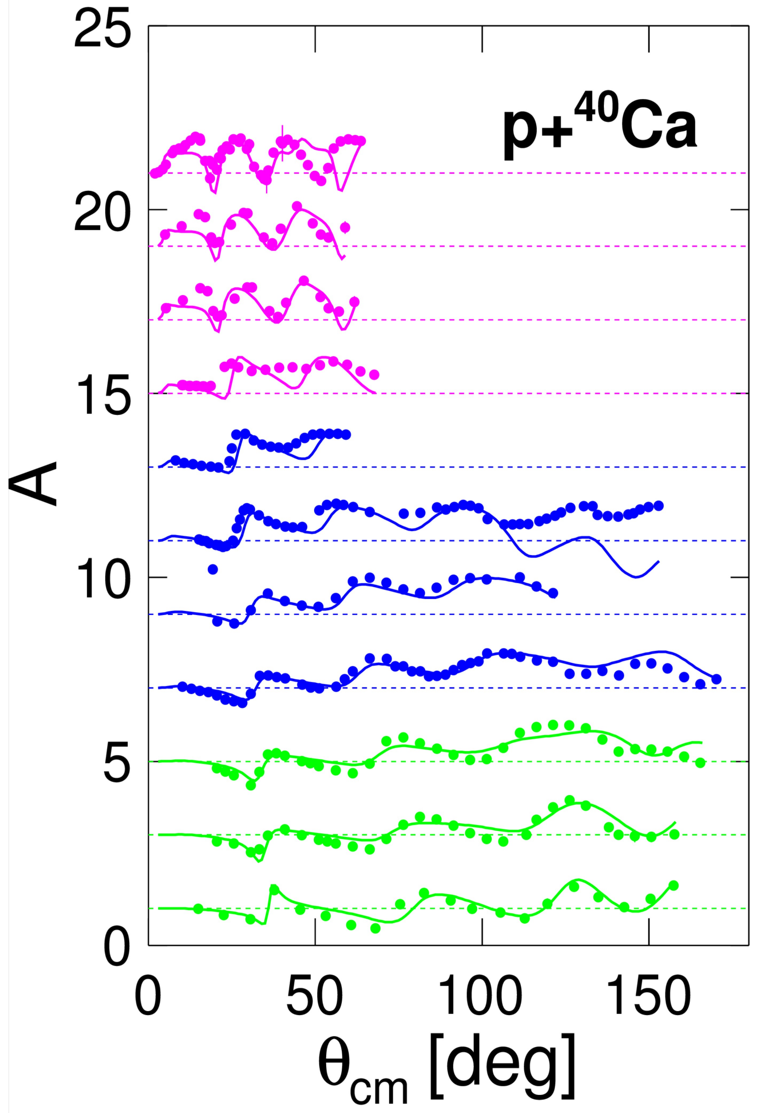
^{40}Ca Cross section



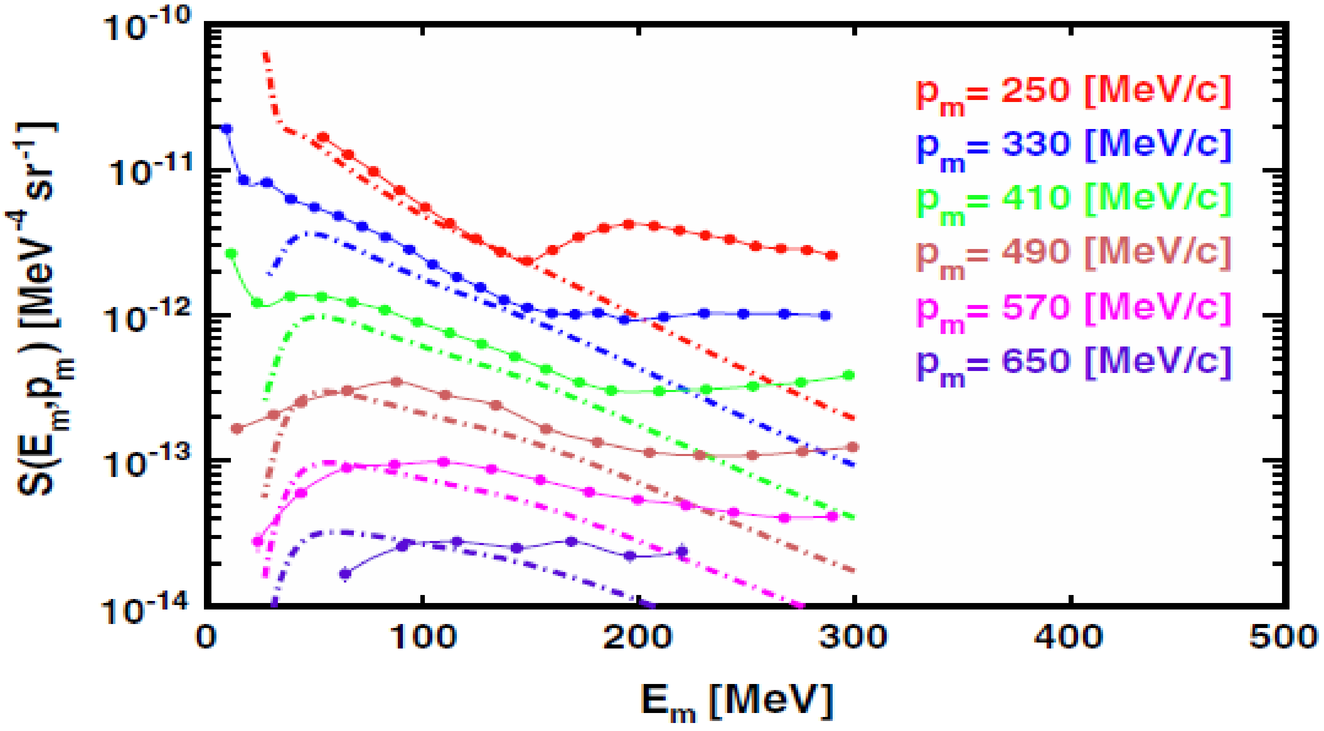
PRL **112**, 162503(2014)

^{40}Ca Cross sections and analyzing power





The hole spectral function for high momenta



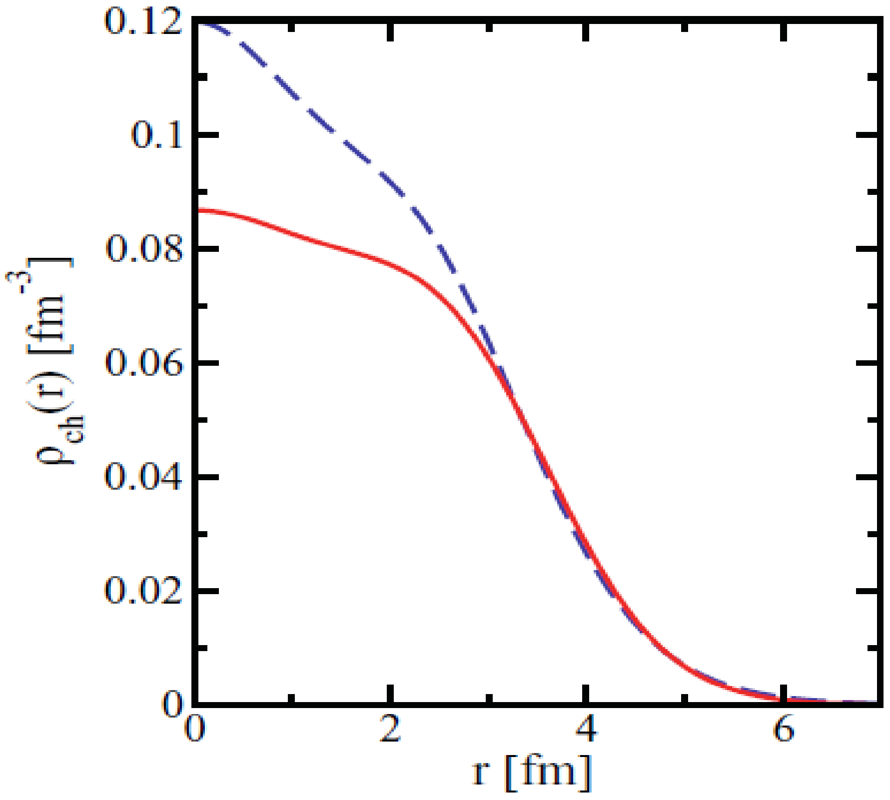
Data:(dotted-line)
D. Rohe, Habilitationsschrift (University of Basel, Basel,2004)

Nonlocal-DOM:(dashed-dotted)

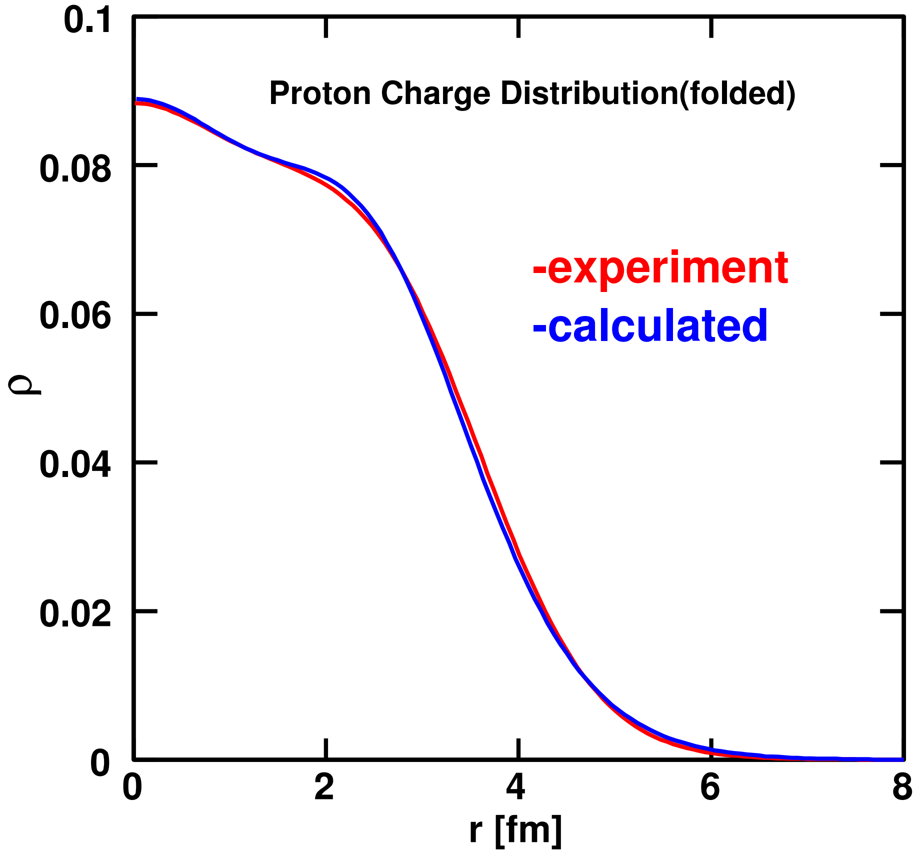
$$S_h(E_m, p_m) = \sum_n \delta(E_m - E_0^N - E_n^{N-1}) \; |\langle \Psi_n^{N-1} | a_{p_m} | \Psi_0^N \rangle|^2$$

^{40}Ca Charge Density

Local DOM



NonLocal DOM



PRC **82**, 054306(2010)

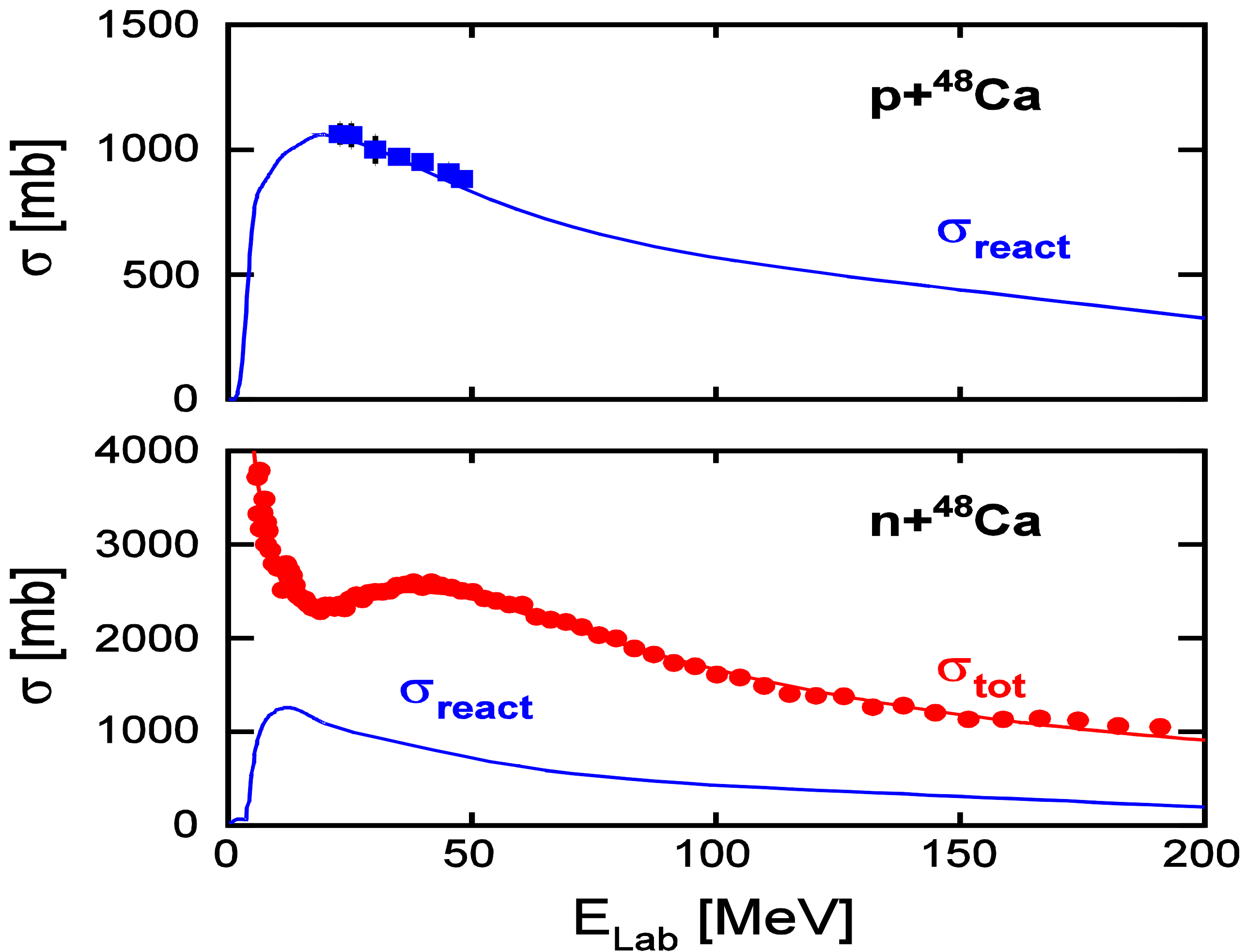
PRL **112**, 162503(2014)

^{48}Ca Cross section

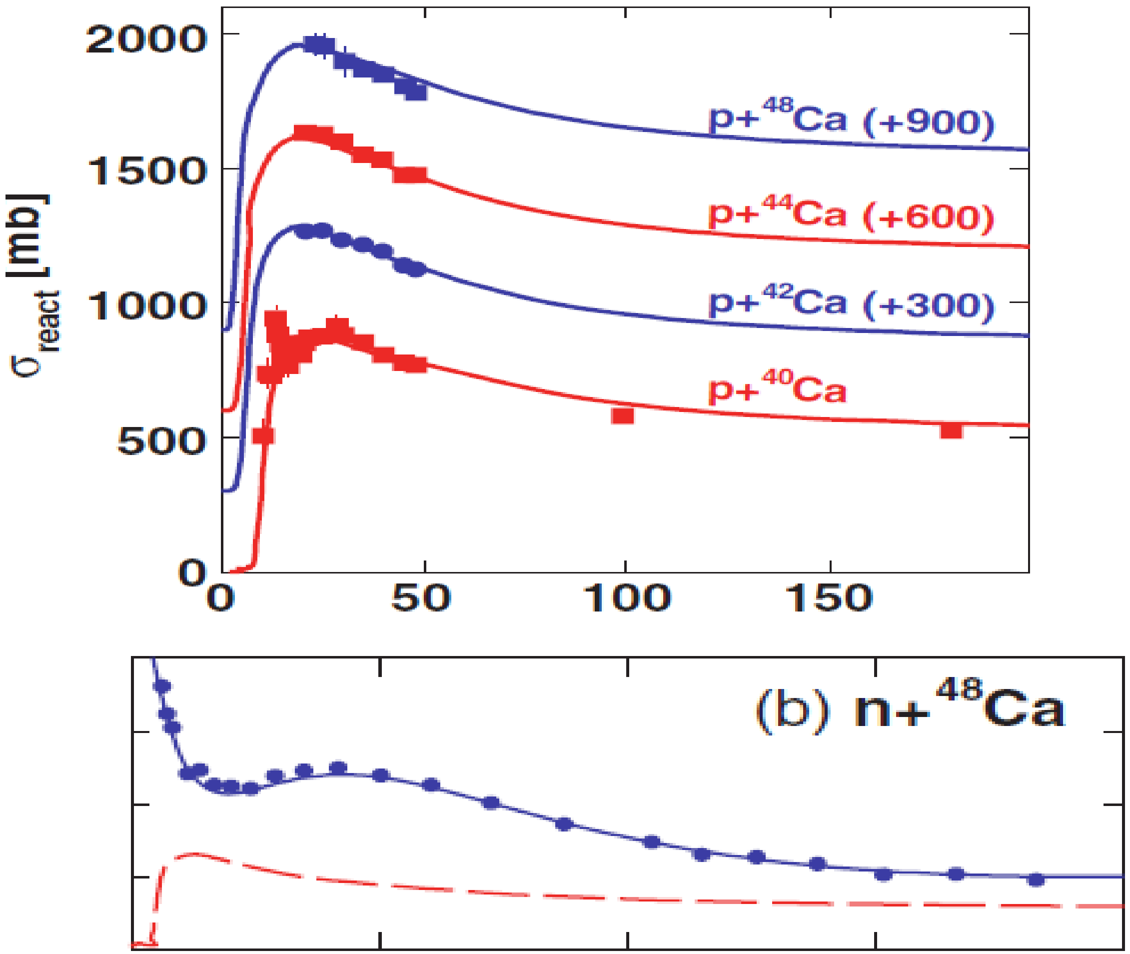
- Including Asymmetry terms proportional to $$\frac{N-Z}{A}$$
- All the parameters kept fixed except the radii (comparing to ^{40}Ca)

^{48}Ca Cross sections

Nonlocal



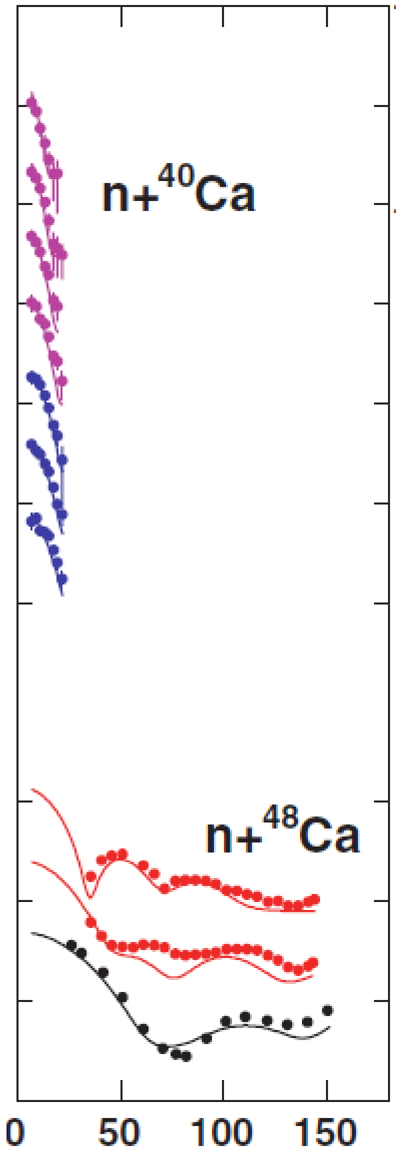
local



PRC **83,** 064605 (2011)

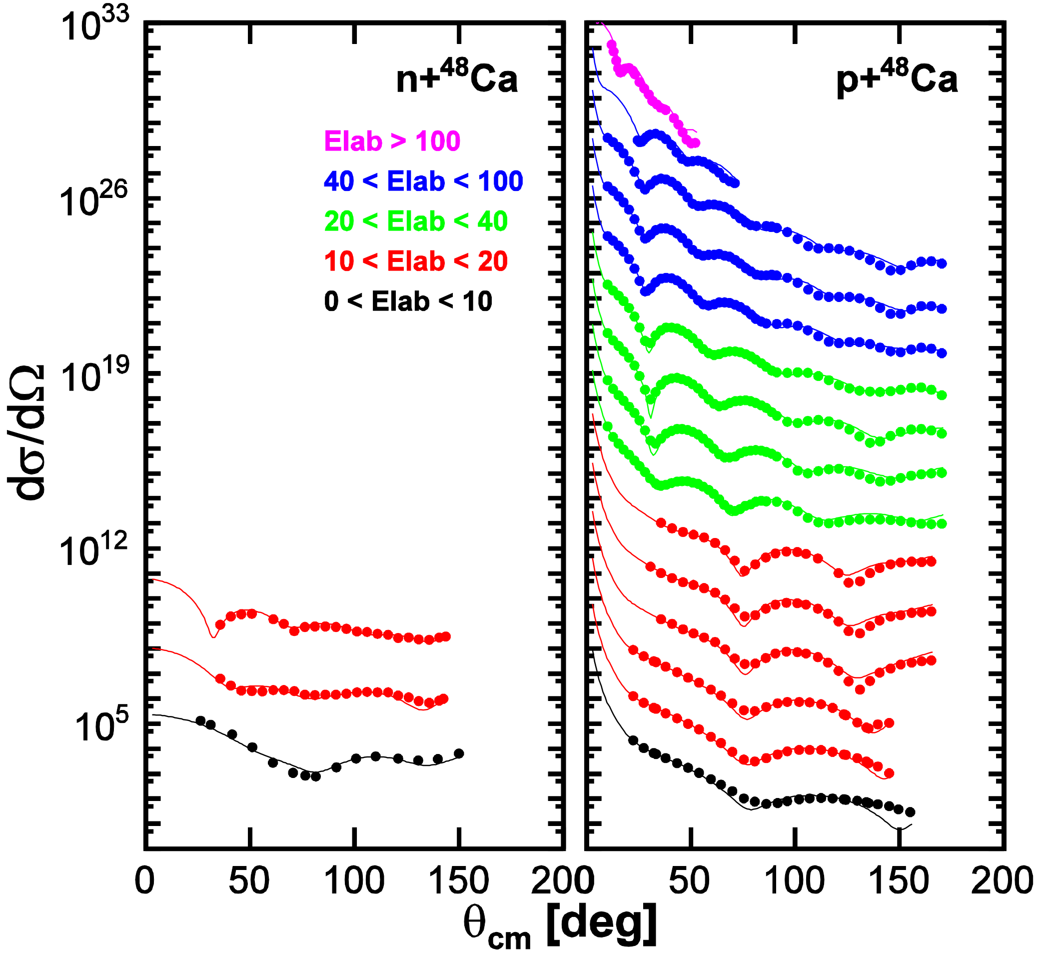
^{48}Ca Cross sections

local

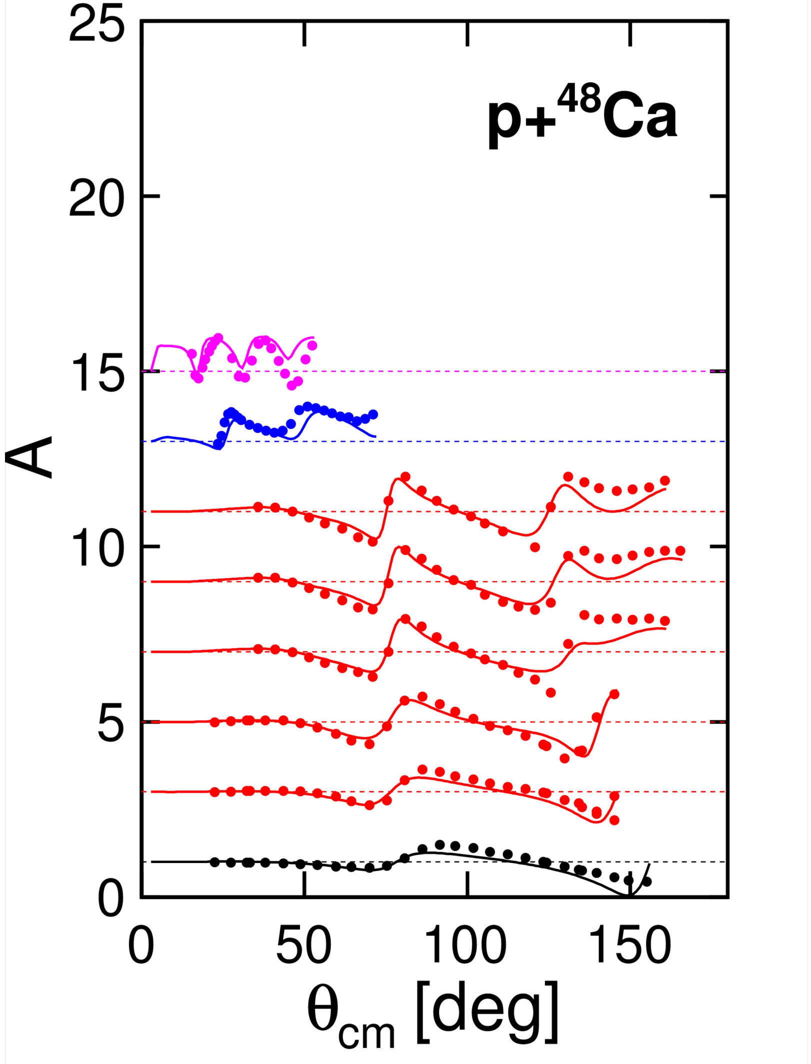


PRC **83,** 064605 (2011)

Non local



PRL **112**, 162503(2014)



Spectroscopic Factors

protons

	40Ca	**48Ca**
1s12	0.73	0.63
0d32	0.76	0.69
0f72	0.73	0.63

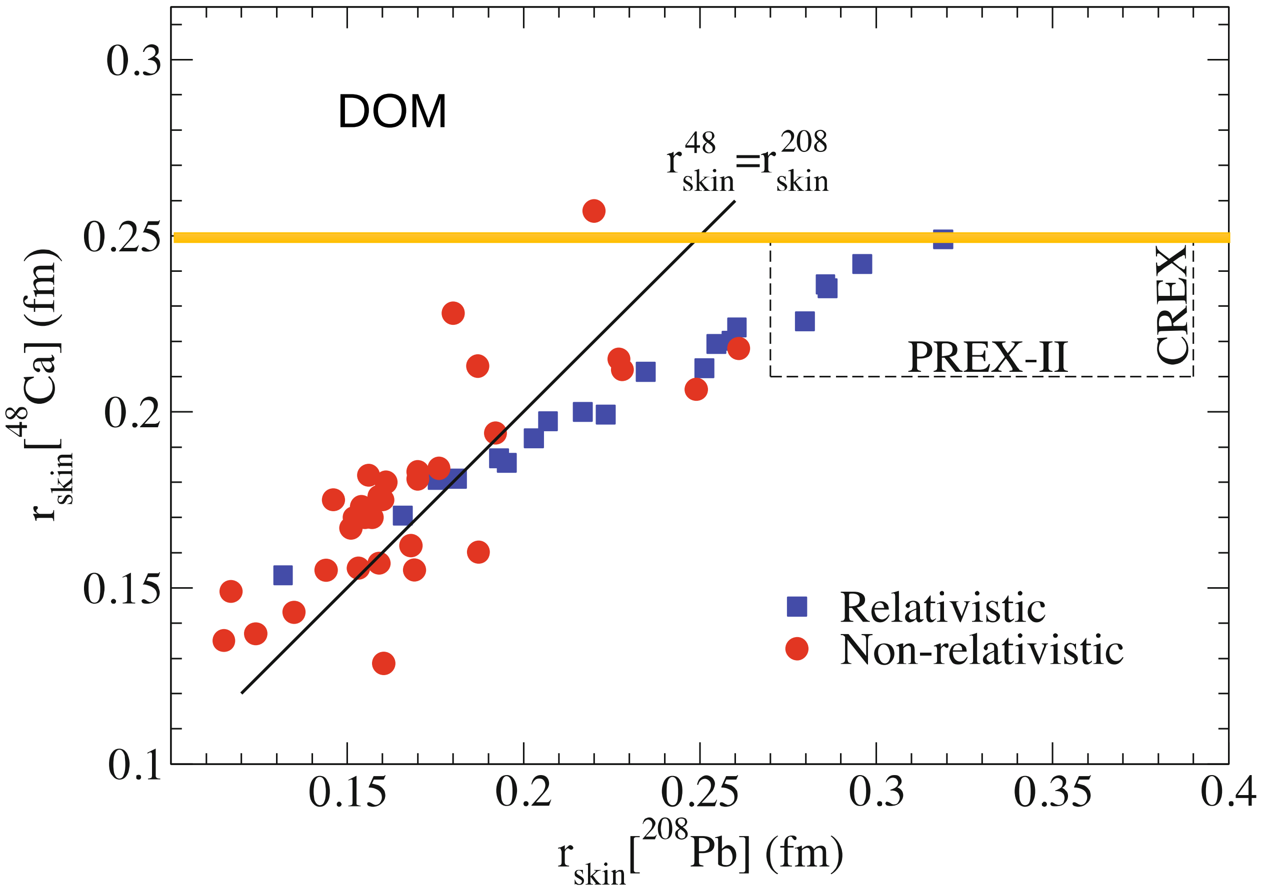
neutrons

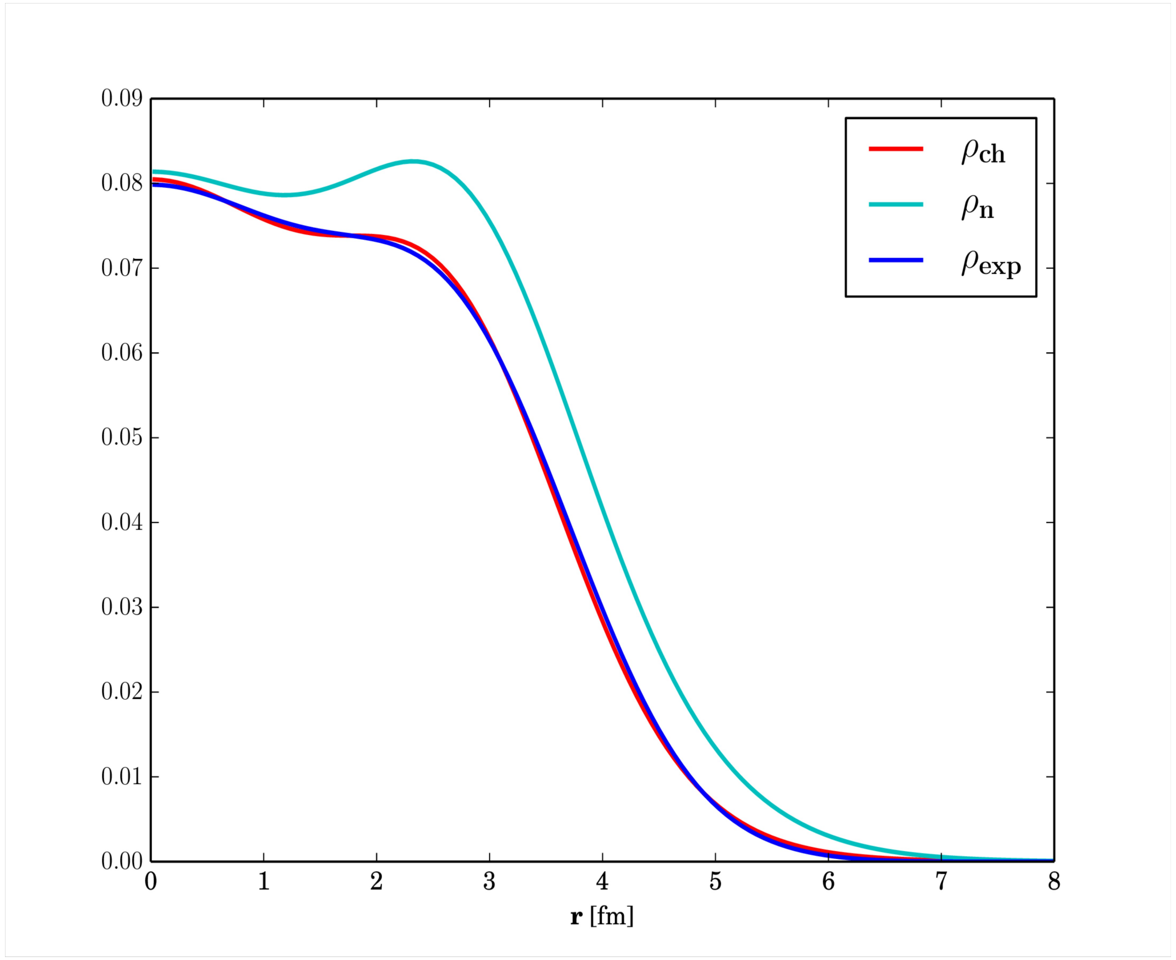
	40Ca	**48Ca**
1s12	0.76	0.80
0d32	0.78	0.77
0f72	0.71	0.80

Weak charge

- The electron interacts with the nucleus by exchanging either a photon or Z0 boson.
- Z0 boson has a much larger coupling to the neutron than protons.

^{48}Ca Charge Density





Eur. Phys. J. A (2014)
C.J. Horowitz, K.S. Kumar, and R. Michaels

Spectral Function

$$S^p_{\ell j}(k,k';E) = \frac{i}{2\pi}\left[G^p_{\ell j}(k,k';E^+) - G^p_{\ell j}(k,k';E^-)\right]$$

$$G^p_{\ell j}(k,k';E^\pm) = \sum_n \frac{\phi^{n+}_{\ell j}(k)\left[\phi^{n+}_{\ell j}(k')\right]^*}{E - E^{*A+1}_n \pm i\eta} + \sum_c \int_{T_c}^{\infty} dE' \, \frac{\chi^{cE'}_{\ell j}(k)\left[\chi^{cE'}_{\ell j}(k')\right]^*}{E - E' \pm i\eta}$$

$$\phi^{n+}_{\ell j}(k) = \langle \Psi^A_0 | a_{k\ell j} | \Psi^{A+1}_n \rangle \qquad \chi^{cE}_{\ell j}(k) = \langle \Psi^A_0 | a_{k\ell j} | \Psi^{A+1}_{cE} \rangle$$

Spectral Function

$$S^p_{\ell j}(r,r';E) = \sum_c \chi^{cE}_{\ell j}(r)\left[\chi^{cE}_{\ell j}(r')\right]^*$$

$$\frac{k^2}{2m}\phi^n_{\ell j}(k) + \int dq\, q^2 \ \mathrm{Re}\ \Sigma^*_{\ell j}(k,q;\varepsilon_n)\phi^n_{\ell j}(q) = \varepsilon_n \phi^n_{\ell j}(k)$$

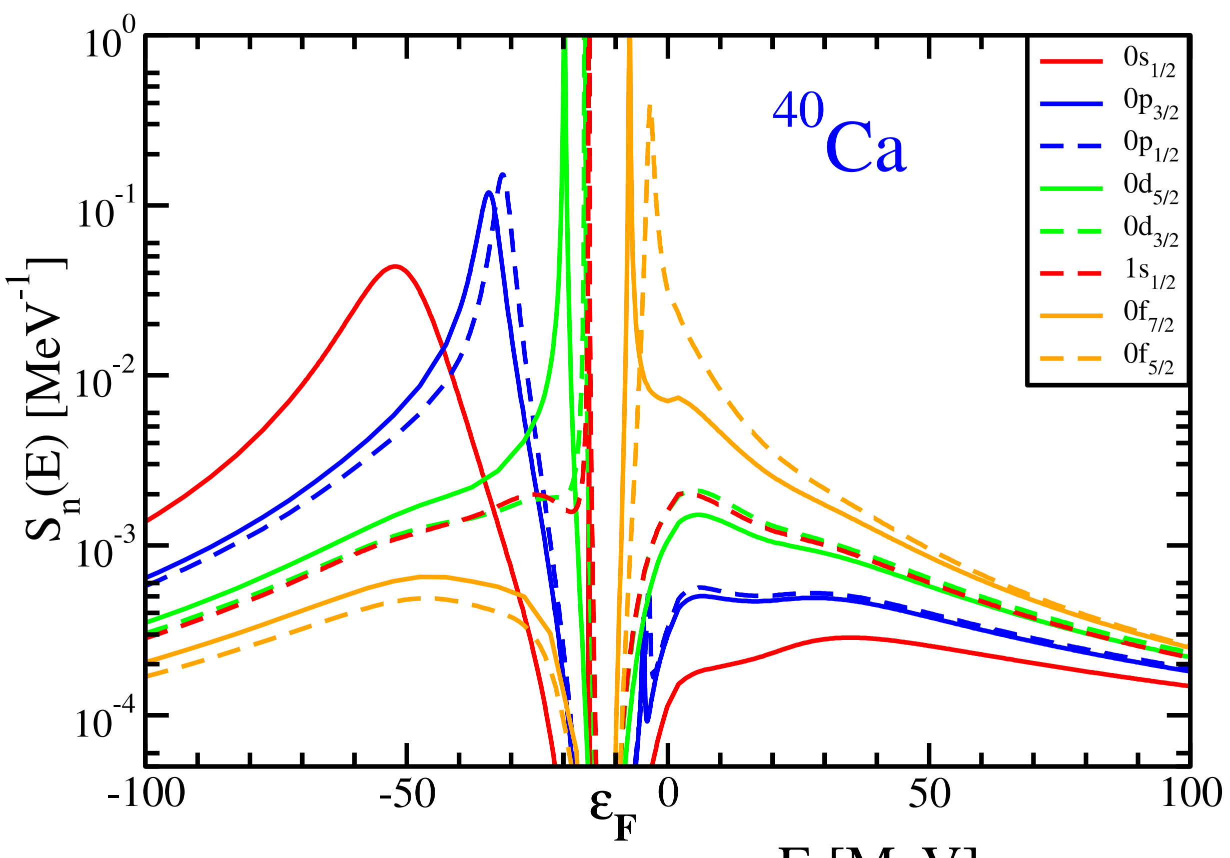
$$S^{n-}_{\ell j}(E) = \int dr r^2 \int dr' r'^2 \phi^{n-}_{\ell j}(r) S^h_{\ell j}(r,r';E)\phi^{n-}_{\ell j}(r'),$$

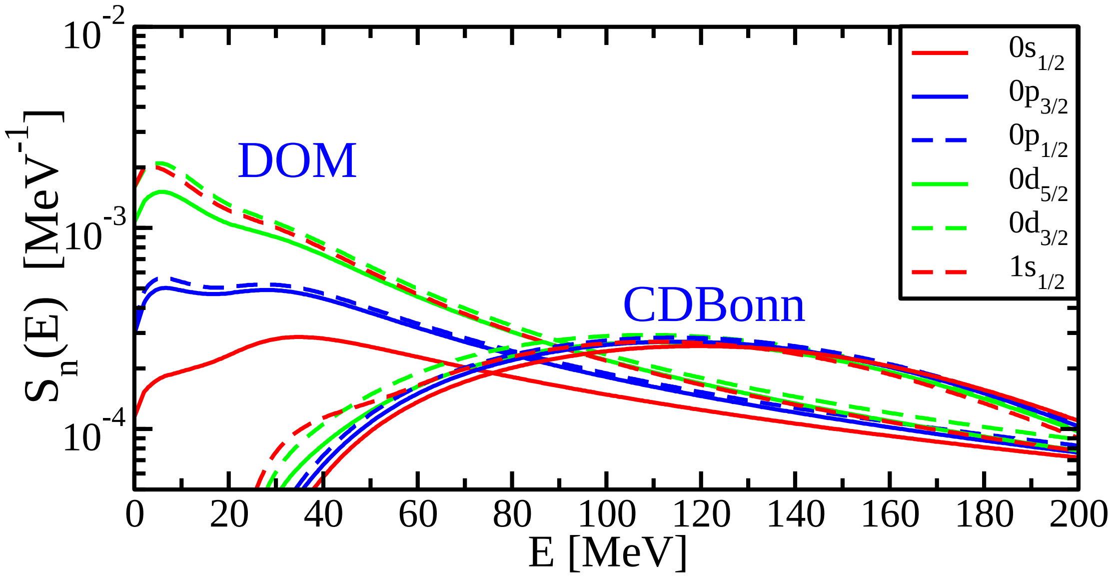
$$S^{n+}_{\ell j}(E) = \int dr r^2 \int dr' r'^2 \phi^{n-}_{\ell j}(r) S^p_{\ell j}(r,r';E)\phi^{n-}_{\ell j}(r'),$$

In Practice $\longrightarrow$ $$S^p_{\ell j}(k,k';E) = \frac{i}{2\pi}\left[G^p_{\ell j}(k,k';E^+) - G^p_{\ell j}(k,k';E^-)\right]$$

$$S^p_{\ell j}(r,r';E) = \frac{2}{\pi}\int dk k^2 \int dk' k'^2 j_\ell(kr) S^p_{\ell j}(k,k';E) j_\ell(k'r'),$$

Spectral Strength



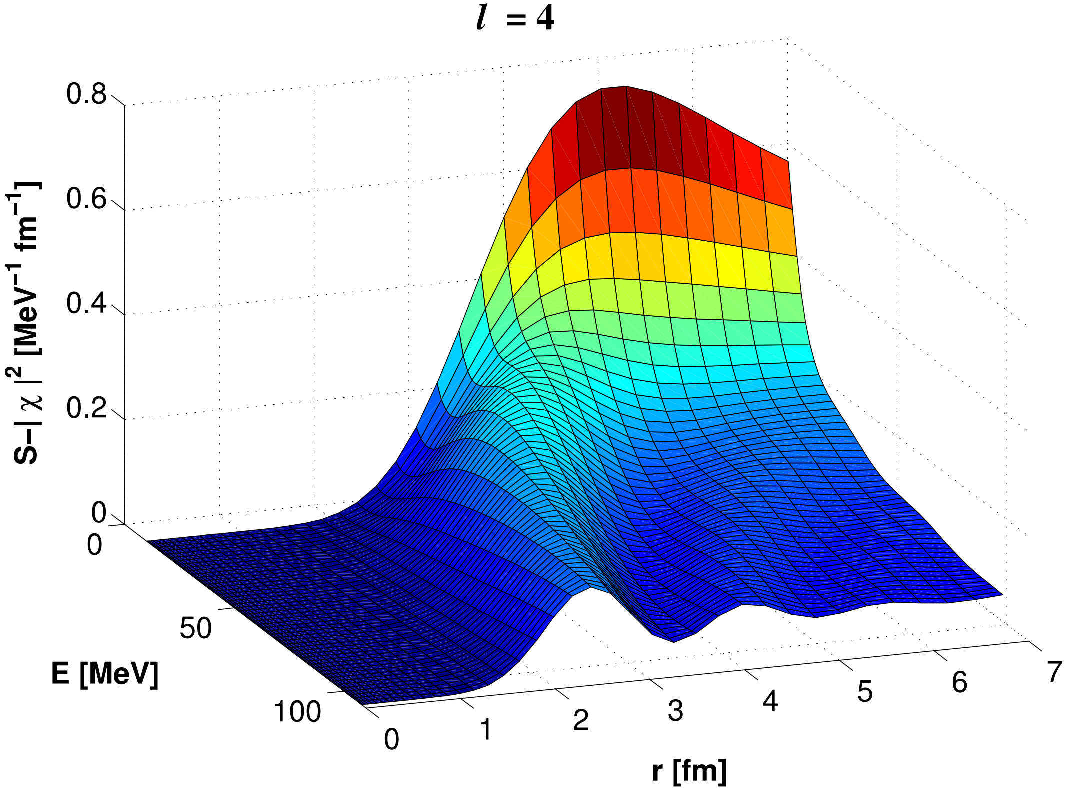


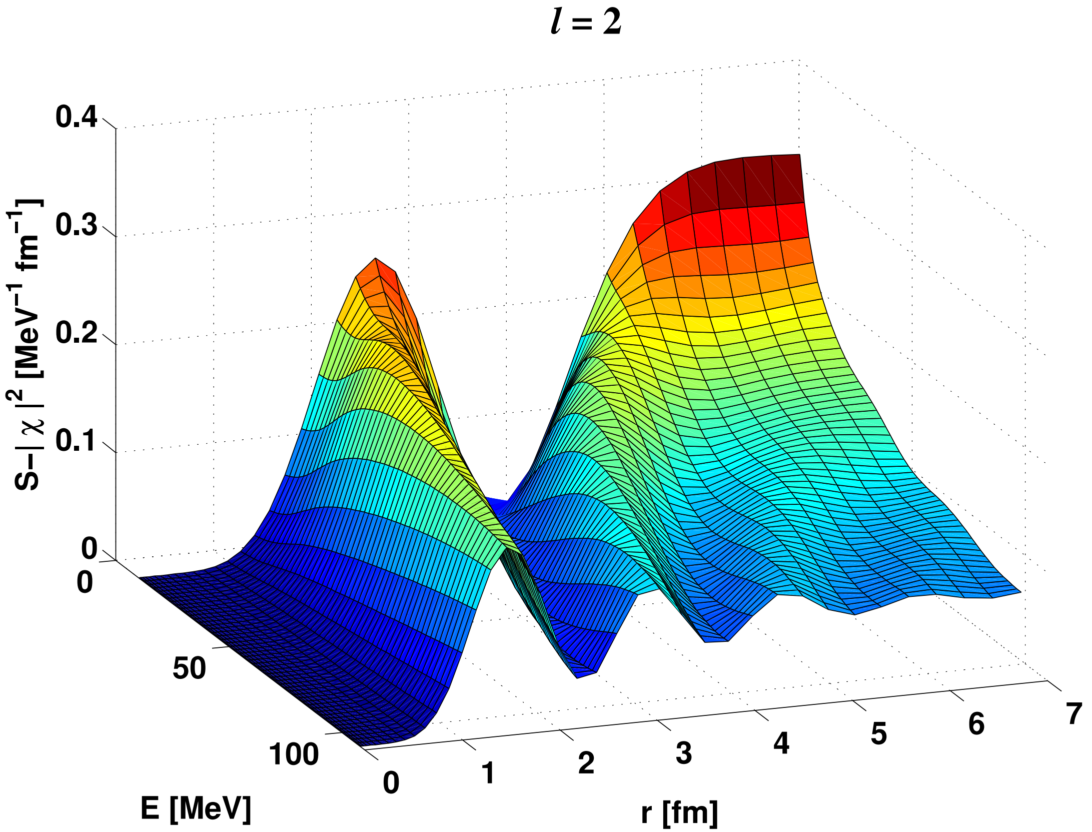
orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0,200]$ DOM	$n_{n\ell j}+d_{n\ell j}[\varepsilon_F,200]$ DOM	$d_{n\ell j}[0,200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$0p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

PHYSICAL REVIEW C 90, 061603(R) (2014)

Spectral Function

$$\chi_{\ell j}^{cE}(k) = \langle \Psi_0^A | a_{k\ell j} | \Psi_{cE}^{A+1} \rangle \qquad \chi_{\ell j}^{elE}(r) = \left[\frac{2mk_0}{\pi\hbar^2}\right]^{1/2} \left\{ j_\ell(k_0 r) + \int dk k^2 j_\ell(kr) G^{(0)}(k;E) \Sigma_{\ell j}(k,k_0;E) \right\}$$





PHYSICAL REVIEW C 90, 061603(R) (2014)

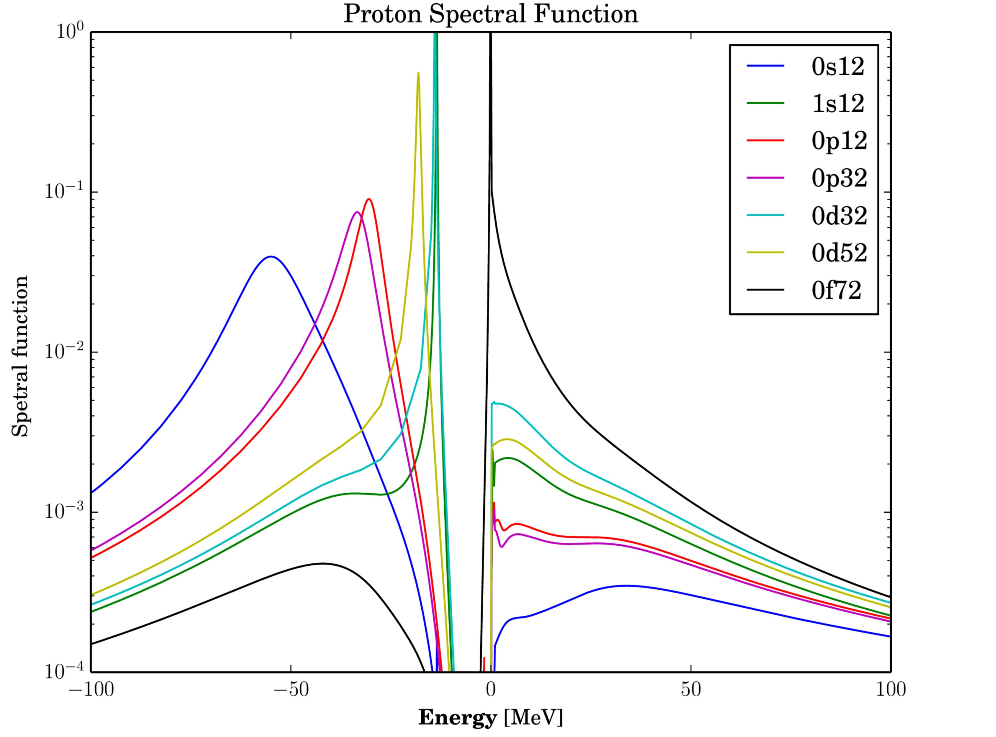
Spectral Function

- Screened Coulomb

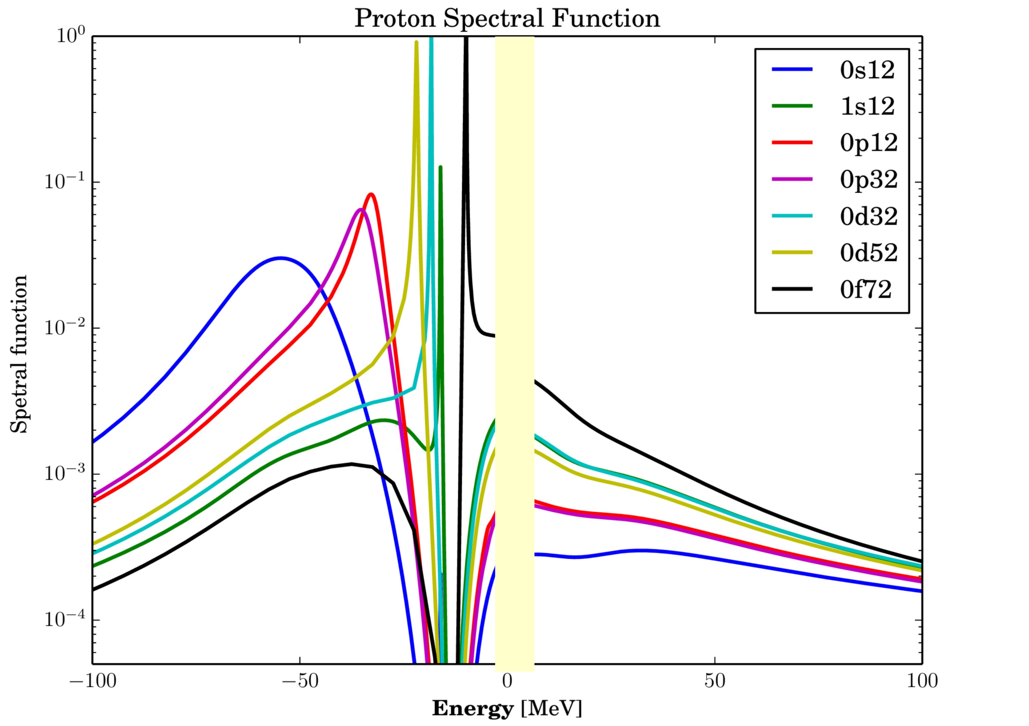
$$w_R(r) = w(r)e^{-(r/R)^n}$$

Phys. Rev. C 41, 2615 (1990)

40Ca,p



48Ca,p



0s12	0p12	0p32		0f72
0.94	0.93	0.95		0.95

sum rule

Conclusion :

- According to the results, Nonlocal DOM is a reliable candidate to study nuclear properties.

-Nonlocality and Dispersion corrections playing an important role to get the physics of the system correctly.