

Present status of perturbative calculation of effective shell-model hamiltonians

Nunzio Itaco

Università di Napoli Federico II
Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

International Collaborations in Nuclear Theory:
Theory for open-shell nuclei near the limits of stability
May 11-29, 2015
Michigan State University and FRIB/NSCL



What is a realistic effective shell-model hamiltonian ?



An example: ^{19}F

^{19}F



protons



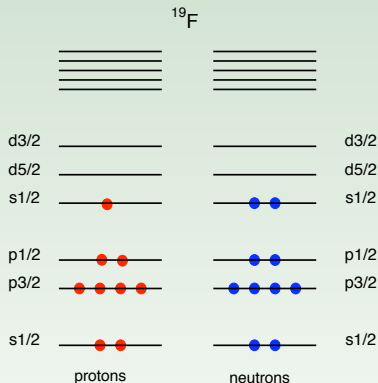
neutrons

- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.



An example: ^{19}F

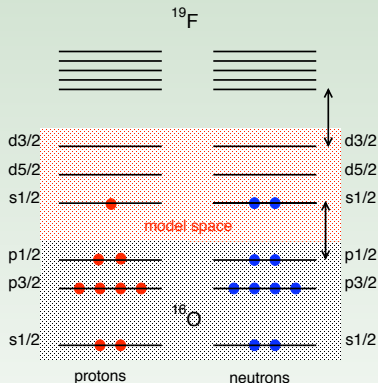


- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.



An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.



Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian



Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian



Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian



Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian



Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian



Workflow for a realistic shell-model calculation

- 1 Choose a realistic NN potential (NNN)
- 2 Determine the model space better tailored to study the system under investigation
- 3 Derive the effective shell-model hamiltonian by way of a many-body theory
- 4 Calculate the physical observables (energies, e.m. transition probabilities, ...)



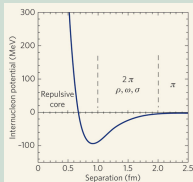
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials

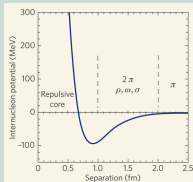
Strong short-range repulsion



Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
 - V_{low-k} (Lee-Suzuki or SRG)



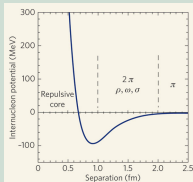
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
 - V_{low-k} (Lee-Suzuki or SRG)
 - chiral potentials rooted in EFT

Strong short-range repulsion



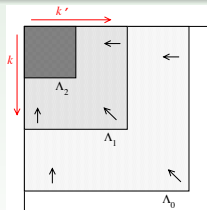
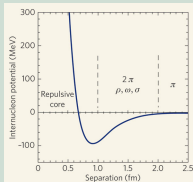
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
 - V_{low-k} (Lee-Suzuki or SRG)
 - chiral potentials rooted in EFT

Strong short-range repulsion



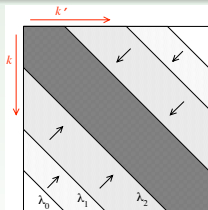
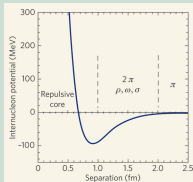
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
 - V_{low-k} (Lee-Suzuki or SRG)
 - chiral potentials rooted in EFT

Strong short-range repulsion



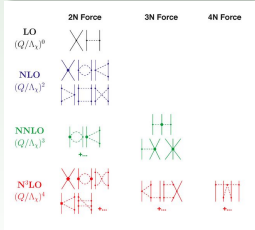
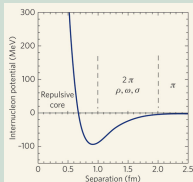
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2 / datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
 - V_{low-k} (Lee-Suzuki or SRG)
 - chiral potentials rooted in EFT

Strong short-range repulsion



The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

with

$$H = H_0 + H_1 = \sum_{i=1}^A (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_i)$$

Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \dots a_n^\dagger]_i |c\rangle \Rightarrow P = \sum_{i=1}^d |\Phi_i\rangle \langle \Phi_i|$$

Model-space eigenvalue problem

$$H_{\text{eff}} P |\Psi_\alpha\rangle = E_\alpha P |\Psi_\alpha\rangle$$



The shell-model effective hamiltonian

$$\begin{pmatrix} PHP & PHQ \\ \hline QHP & QHQ \end{pmatrix} \begin{matrix} \mathcal{H} = X^{-1}HX \\ \Rightarrow \\ QHP = 0 \end{matrix} \begin{pmatrix} PHP & PHQ \\ \hline 0 & QHQ \end{pmatrix}$$

$$H_{\text{eff}} = PHP$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \begin{pmatrix} 0 & 0 \\ Q\omega P & 0 \end{pmatrix}$



The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \mathcal{H} = X^{-1}HX \Rightarrow \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$
$$QHP = 0$$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$



The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \mathcal{H} = X^{-1}HX \Rightarrow \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$Q\mathcal{H}P = 0$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$



The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \mathcal{H} = X^{-1}HX \Rightarrow \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$
$$QHP = 0$$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$



The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \mathcal{H} = X^{-1}HX \Rightarrow \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$QHP = 0$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$



Folded-diagram expansion

\hat{Q} -box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

\Rightarrow Recursive equation for H_{eff} \Rightarrow iterative techniques
(Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \dots,$$

generalized folding



The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

The \hat{Q} -box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the \hat{Q} -box



The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

The \hat{Q} -box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the \hat{Q} -box



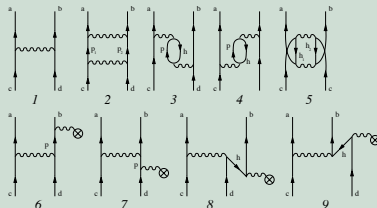
The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

The \hat{Q} -box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the \hat{Q} -box



The perturbative approach to the shell-model H^{eff}

- H^{eff} for systems with one and two valence nucleons
- \hat{Q} -box \Rightarrow Goldstone diagrams up to third order in V_{NN} (up to 2p-2h core excitations)
- Padè approximant [2|1] of the \hat{Q} -box

$$[2|1] = V_{Qbox}^0 + V_{Qbox}^1 + V_{Qbox}^2 (1 - (V_{Qbox}^2)^{-1} V_{Qbox}^3)^{-1} ,$$



The perturbative approach to the shell-model H^{eff}

- H^{eff} for systems with one and two valence nucleons
- \hat{Q} -box \Rightarrow Goldstone diagrams up to third order in V_{NN} (up to 2p-2h core excitations)
- Padè approximant [2|1] of the \hat{Q} -box

$$[2|1] = V_{Qbox}^0 + V_{Qbox}^1 + V_{Qbox}^2 (1 - (V_{Qbox}^2)^{-1} V_{Qbox}^3)^{-1} ,$$



The perturbative approach to the shell-model H^{eff}

- H^{eff} for systems with one and two valence nucleons
- \hat{Q} -box \Rightarrow Goldstone diagrams up to third order in V_{NN} (up to 2p-2h core excitations)
- Padè approximant $[2|1]$ of the \hat{Q} -box

$$[2|1] = V_{Qbox}^0 + V_{Qbox}^1 + V_{Qbox}^2 (1 - (V_{Qbox}^2)^{-1} V_{Qbox}^3)^{-1} ,$$



Test case: p -shell nuclei

- L. Coraggio (INFN)
- A. Covello (UNINA and INFN)
- A. Gargano (INFN)
- T. T. S. Kuo (SUNY at Stony Brook)
- N. I. (UNINA and INFN)

*L. Coraggio, A. Covello, A. Gargano, N. I., and T. T. S. Kuo, Ann. Phys. **327** 2125 (2012)*

First, some convergence checks !



Test case: p -shell nuclei

- $V_{NN} \Rightarrow$ chiral N^3LO potential by Entem & Machleidt (smooth cutoff $\simeq 2.5 \text{ fm}^{-1}$)
- H_{eff} for two valence nucleons outside ${}^4\text{He}$
- Single-particle energies and residual two-body interaction are derived from the theory. **No empirical input**

First, some convergence checks !



Test case: p -shell nuclei

- $V_{NN} \Rightarrow$ chiral N^3LO potential by Entem & Machleidt (smooth cutoff $\simeq 2.5 \text{ fm}^{-1}$)
- H_{eff} for two valence nucleons outside ${}^4\text{He}$
- Single-particle energies and residual two-body interaction are derived from the theory. **No empirical input**

First, some convergence checks !



Convergence checks

The intermediate-state space Q

Q -space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value E_{max} are disregarded

$$|\epsilon_0 - QH_0Q| \leq E_{max} = N_{max} \hbar\omega$$

${}^6\text{Li}$ yrast states

results quite stable for
 $N_{max} \geq 20$



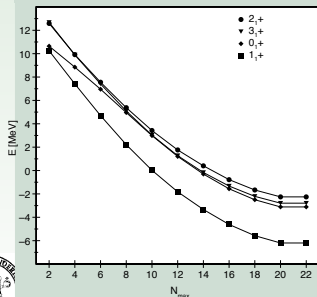
Convergence checks

The intermediate-state space Q

Q -space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value E_{max} are disregarded

$$|\epsilon_0 - QH_0Q| \leq E_{max} = N_{max} \hbar\omega$$

${}^6\text{Li}$ yrast states



results quite stable for
 $N_{max} \geq 20$



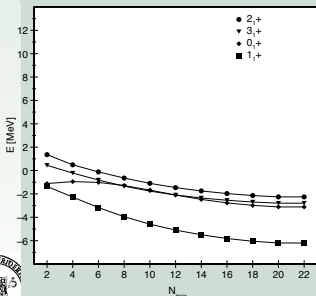
Convergence checks

The intermediate-state space Q

Q -space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value E_{max} are disregarded

$$|\epsilon - QH_0Q| \leq E_{max} = N_{max}\hbar\omega$$

${}^6\text{Li}$ yrast states



results much more stable



Order-by-order convergence

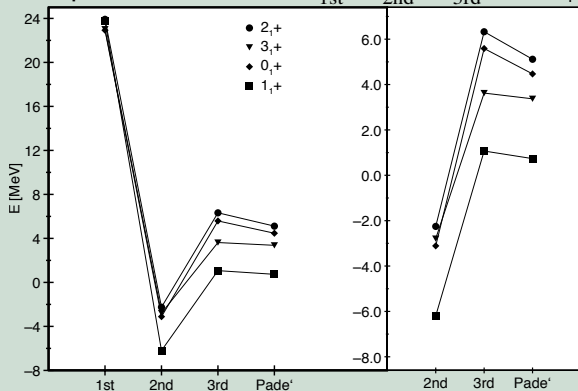
Compare results from H_{1st}^{eff} , H_{2nd}^{eff} , H_{3rd}^{eff} and $H_{Padè}^{eff}$



Convergence checks

Order-by-order convergence

Compare results from H_{1st}^{eff} , H_{2nd}^{eff} , H_{3rd}^{eff} and $H_{Padè}^{eff}$



Convergence checks

Dependence on $\hbar\omega$

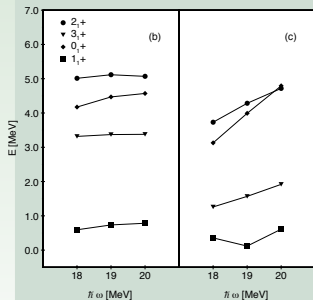
Auxiliary potential $U \Rightarrow$ harmonic oscillator potential



Convergence checks

Dependence on $\hbar\omega$

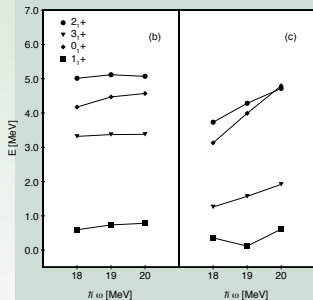
Auxiliary potential $U \Rightarrow$ harmonic oscillator potential



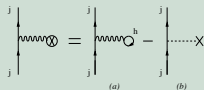
Convergence checks

Dependence on $\hbar\omega$

Auxiliary potential $U \Rightarrow$ harmonic oscillator potential



HF-insertions



- zero in a self-consistent basis
- neglected in most applications
- disregard of HF-insertions introduces relevant dependence on $\hbar\omega$



Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C 69, 014311 (2004)

P. Navrátil *et al.*, Phys. Rev. Lett. 99, 042501 (2007)



Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)



Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)



Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)



Benchmark calculation

To compare our results with NCSM we need to start from a translationally invariant Hamiltonian

$$\begin{aligned} H_{int} &= \left(1 - \frac{1}{A}\right) \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i < j=1}^A \left(V_{ij}^{NN} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) = \\ &= \left[\sum_{i=1}^A \left(\frac{p_i^2}{2m} + U_i \right) \right] + \left[\sum_{i < j=1}^A \left(V_{ij}^{NN} - U_i - \frac{p_i^2}{2mA} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) \right] \end{aligned}$$

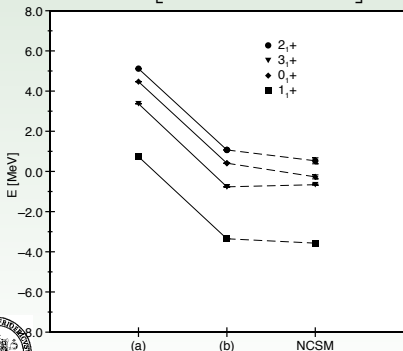


Benchmark calculation

To compare our results with NCSM we need to start from a translationally invariant Hamiltonian

$$H_{int} = \left(1 - \frac{1}{A}\right) \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j=1}^A \left(V_{ij}^{NN} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) =$$

$$= \left[\sum_{i=1}^A \left(\frac{p_i^2}{2m} + U_i \right) \right] + \left[\sum_{i<j=1}^A \left(V_{ij}^{NN} - U_i - \frac{p_i^2}{2mA} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) \right]$$



(a) not translationally invariant Hamiltonian

(b) purely intrinsic hamiltonian



Benchmark calculation

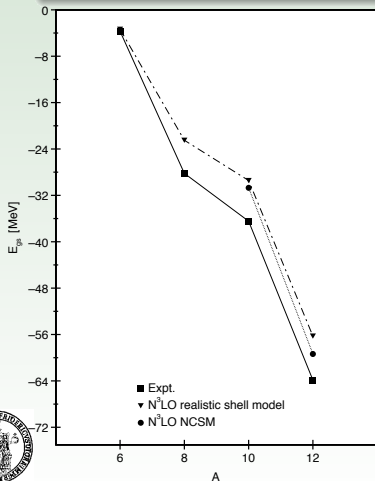
Remark

H^{eff} derived for 2 valence nucleon systems \Rightarrow 3-, 4-, .. n -body components are neglected



Remark

H^{eff} derived for 2 valence nucleon systems \Rightarrow 3-, 4-, .. n -body components are neglected



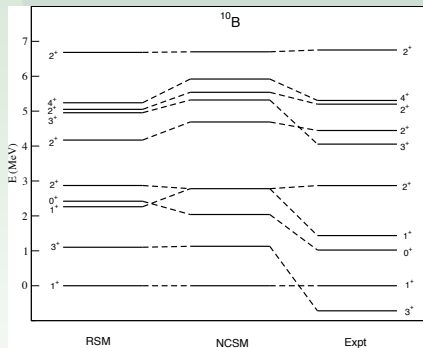
- ground-state energies for $N = Z$ nuclei
- discrepancy grows with the number of valence nucleons



^{10}B relative spectrum



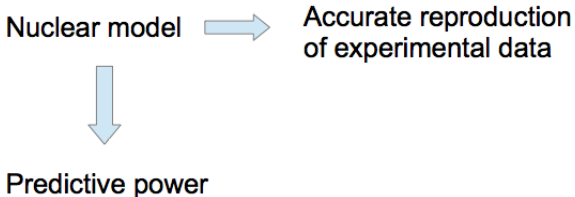
^{10}B relative spectrum



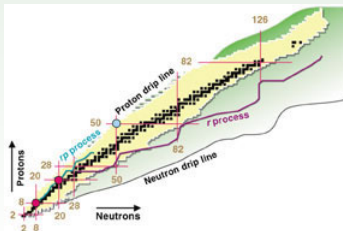
- discrepancy ≤ 1 MeV
- minor role of many-body correlations



Nuclear models and predictive power



RIBs & advances in detection techniques \Rightarrow unknown structure of nuclei towards the drip lines



Realistic shell-model calculations

realistic shell-model calculations in different mass regions



results in good agreement with experimental data

Can realistic shell-model calculations be predictive ?
few selected examples



Realistic shell-model calculations

realistic shell-model calculations in different mass regions



results in good agreement with experimental data

Can realistic shell-model calculations be predictive ?

few selected examples



Realistic shell-model calculations

realistic shell-model calculations in different mass regions



results in good agreement with experimental data

Can realistic shell-model calculations be predictive ?
few selected examples



Few selected physics cases

- neutron-deficient tin isotopes
- Sn isotopes beyond $N = 82$
- heavy calcium isotopes

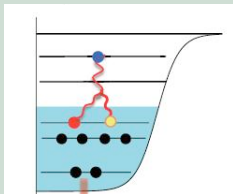
Single-particle energies from the experiment \Rightarrow reduced role of $3N$ force



Few selected physics cases

- neutron-deficient tin isotopes
- Sn isotopes beyond $N = 82$
- heavy calcium isotopes

Single-particle energies from the experiment \Rightarrow reduced role of $3N$ force



Neutron-deficient Sn isotopes

^{100}Sn is the heaviest particle-bound doubly-magic nucleus with $N = Z$



Neutron-deficient Sn isotopes

^{100}Sn is the heaviest particle-bound doubly-magic nucleus with $N = Z$

PHYSICAL REVIEW C

VOLUME 54, NUMBER 4

OCTOBER 1996

Realistic shell-model calculations for neutron deficient Sn isotopes

F. Andreozzi,¹ L. Coraggio,¹ A. Covello,¹ A. Gargano,¹ T. T. S. Kuo,² Z. B. Li,² and A. Porrino¹

¹*Dipartimento di Scienze Fisiche, Università di Napoli Federico II
and Istituto Nazionale di Fisica Nucleare, Mostra d'Oltremare, Pad. 20, 80125 Napoli, Italy*

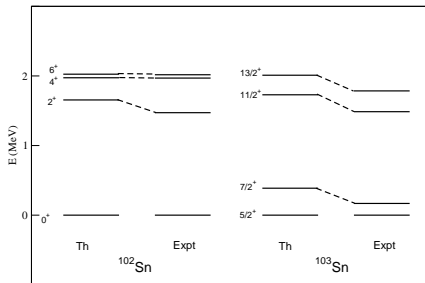
²*Department of Physics, SUNY, Stony Brook, New York 11794*

(Received 27 November 1995; revised manuscript received 28 June 1996)

- ⇒ $^{102-105}\text{Sn}$ studied starting from Bonn A NN potential
- ⇒ $g_{7/2}dsh_{11/2}$ model space with ^{100}Sn inert core
- ⇒ SP energies from analysis of low-energy spectra of heavier tin isotopes ($105 \leq A \leq 111$)
- ⇒ predictions for the (at that time) unknown spectra of $^{102-103}\text{Sn}$



Neutron-deficient Sn isotopes: shell-model results



- very good agreement with experiment
- overestimation of 2^+ energy in $^{102}\text{Sn} \rightarrow Z=50$ cross-shell excitations (see Luigi's talk)



Shell-model study of exotic Sn isotopes with a realistic effective interaction

A Covello^{1,2}, L Coraggio², A Gargano² and N Itaco^{1,2}

¹Dipartimento di Scienze Fisiche, Università di Napoli Federico II,
Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

²Istituto Nazionale di Fisica Nucleare,
Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

- ⇒ shell-model study of Sn isotopes beyond $N = 82$
- ⇒ $V_{\text{low}-k}$ from CD-Bonn NN potential
- ⇒ $h_{9/2} fpi_{13/2}$ model space with ^{132}Sn inert core
- ⇒ SP energies from ^{133}Sn



Shell-model study of exotic Sn isotopes with a realistic effective interaction

A Covello^{1,2}, L Coraggio², A Gargano² and N Itaco^{1,2}

¹Dipartimento di Scienze Fisiche, Università di Napoli Federico II,
Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

²Istituto Nazionale di Fisica Nucleare,
Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

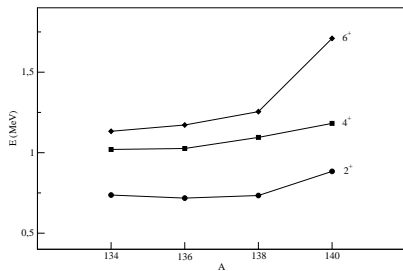
⇒ shell-model study of Sn isotopes beyond $N = 82$

... It is the aim of our study to compare the results of our calculations with the available experimental data and to make predictions for the neighboring heavier isotopes ...



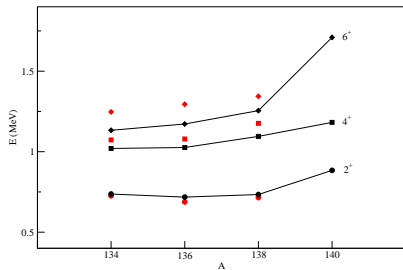
Sn isotopes beyond $N = 82$

Excitation energies of the 2_1^+ , 4_1^+ , and 6_1^+ states in Sn isotopes



Sn isotopes beyond $N = 82$

Excitation energies of the 2_1^+ , 4_1^+ , and 6_1^+ states in Sn isotopes



PRL 113, 132502 (2014)

PHYSICAL REVIEW LETTERS

week ending
26 SEPTEMBER 2014

Yrast 6^+ Seniority Isomers of $^{136,138}\text{Sn}$

G. S. Simpson,^{1,2,3} G. Gey,^{3,4,5} A. Jungclaus,⁶ J. Taprogge,^{6,7,5} S. Nishimura,⁵ K. Sieja,⁸ P. Doornenbal,⁵ G. Lorusso,⁵ P.-A. Söderström,⁵ T. Sumikama,⁹ Z. Y. Xu,¹⁰ H. Baba,⁵ F. Browne,^{11,5} N. Fukuda,⁵ N. Inabe,⁵ T. Isobe,⁵ H. S. Jung,^{12,*} D. Kameda,⁵ G. D. Kim,¹³ Y.-K. Kim,^{13,14} I. Kojouharov,¹⁵ T. Kubo,⁵ N. Kurz,¹⁵ Y. K. Kwon,¹³ Z. Li,¹⁶ H. Sakurai,^{5,10} H. Sakurai,¹⁵ Z. Wang,⁵ T. Wang,⁵ M. Wang,^{17,5} W. Wang,⁵ T. Wang,^{16,5} A. Wang,¹⁸ T. Wang,¹⁹



LETTER

doi:10.1038/nature12226

Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz¹, D. Beck², K. Blaum³, Ch. Borgmann³, M. Breitenfeldt⁴, R. B. Cakirli^{3,5}, S. George¹, F. Herfurth², J. D. Holt^{6,7}, M. Kowalska⁸, S. Kreim^{3,8}, D. Lunney⁹, V. Manea⁹, J. Menéndez^{6,7}, D. Neidherr², M. Rosenbusch¹, L. Schweikhard¹, A. Schwenk^{7,6}, J. Simonis^{6,7}, J. Stanja¹⁰, R. N. Wolf⁸ & K. Zuber¹⁰

- ⇒ first mass measurements of ^{53}Ca and ^{54}Ca
- ⇒ new method of precision mass spectroscopy with ISOLTRAP



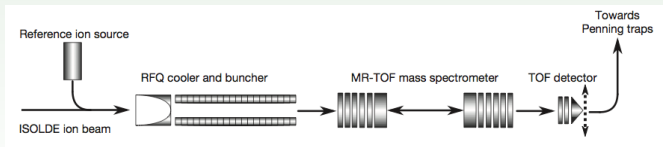
LETTER

doi:10.1038/nature12226

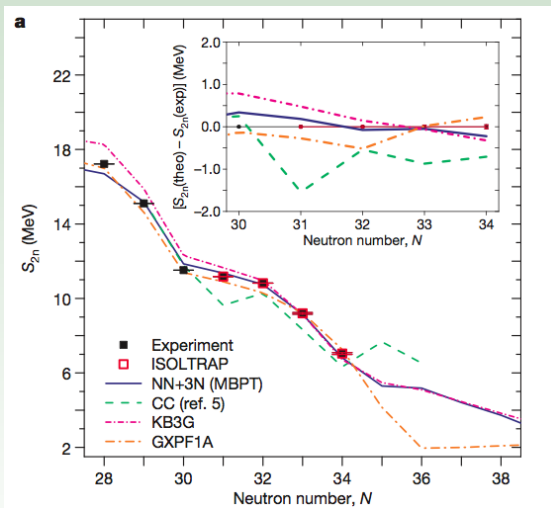
Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz¹, D. Beck², K. Blaum³, Ch. Borgmann³, M. Breitenfeldt⁴, R. B. Cakirli^{3,5}, S. George¹, F. Herfurth², J. D. Holt^{6,7}, M. Kowalska⁸, S. Kreim^{3,8}, D. Lunney⁹, V. Manea⁹, J. Menéndez^{6,7}, D. Neidherr², M. Rosenbusch¹, L. Schweikhard¹, A. Schwenk^{7,6}, J. Simonis^{6,7}, J. Stanja¹⁰, R. N. Wolf¹ & K. Zuber¹⁰

- ⇒ first mass measurements of ^{53}Ca and ^{54}Ca
- ⇒ new method of precision mass spectroscopy with ISOLTRAP



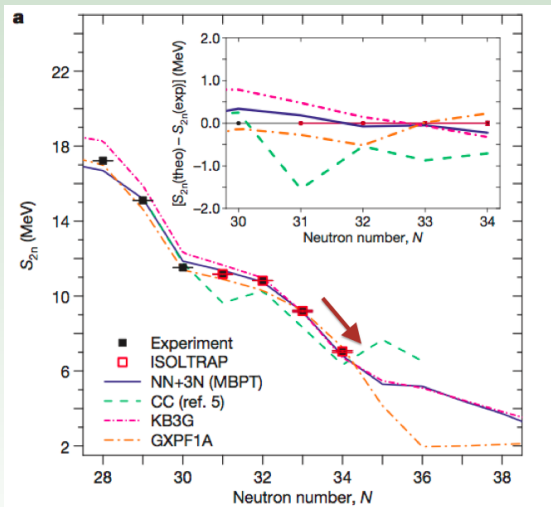
Heavy calcium isotopes



“ ... pronounced decrease in S_{2n} revealed by the new ^{53}Ca and ^{54}Ca ISOLTRAP masses ... ”



Heavy calcium isotopes



“ ... pronounced decrease in S_{2n} revealed by the new ^{53}Ca and ^{54}Ca ISOLTRAP masses ... ”



LETTER

doi:10.1038/nature12522

Evidence for a new nuclear 'magic number' from the level structure of ^{54}Ca

D. Steppenbeck¹, S. Takeuchi², N. Aoi³, P. Doornenbal², M. Matsushita¹, H. Wang², H. Baba², N. Fukuda², S. Go¹, M. Honma⁴, J. Lee², K. Matsui⁵, S. Michimasa¹, T. Motobayashi², D. Nishimura⁶, T. Otsuka^{1,3}, H. Sakurai^{2,5}, Y. Shiga⁷, P.-A. Söderström², T. Sumikama⁸, H. Suzuki², R. Tanuchi⁹, Y. Utsuno⁹, J. J. Valiente-Dobón¹⁰ & K. Yoneda²

⇒ spectroscopic study of ^{54}Ca

⇒ proton knockout reactions involving ^{55}Sc and ^{56}Ti projectiles



LETTER

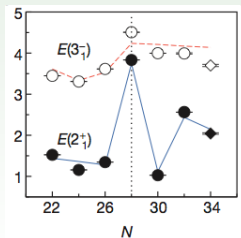
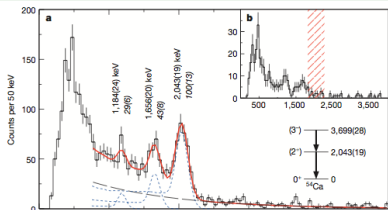
doi:10.1038/nature12522

Evidence for a new nuclear 'magic number' from the level structure of ^{54}Ca

D. Steppenbeck¹, S. Takeuchi², N. Aoi³, P. Doornenbal², M. Matsushita¹, H. Wang², H. Baba², N. Fukuda², S. Go¹, M. Honma⁴, J. Lee², K. Matsui², S. Michimasa¹, T. Motobayashi², D. Nishimura⁶, T. Otsuka^{1,2,5}, H. Sakurai^{2,5}, Y. Shiga⁷, P.-A. Söderström², T. Sumikama⁸, H. Suzuki², R. Tanluchi⁹, Y. Utsuno⁹, J. J. Valiente-Dobón¹⁰ & K. Yoneda²

⇒ spectroscopic study of ^{54}Ca

⇒ proton knockout reactions involving ^{55}Sc and ^{56}Ti projectiles



PHYSICAL REVIEW C **80**, 044311 (2009)

Spectroscopic study of neutron-rich calcium isotopes with a realistic shell-model interaction

L. Coraggio,¹ A. Covello,^{1,2} A. Gargano,¹ and N. Itaco^{1,2}

¹*Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*

²*Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*

(Received 30 July 2009; published 12 October 2009)

- ⇒ shell-model study of neutron-rich calcium isotopes
- ⇒ *fp* model space with ^{40}Ca inert core
- ⇒ predictions for the (at that time) unknown spectra of $^{53-56}\text{Ca}$



PHYSICAL REVIEW C **80**, 044311 (2009)

Spectroscopic study of neutron-rich calcium isotopes with a realistic shell-model interaction

L. Coraggio,¹ A. Covello,^{1,2} A. Gargano,¹ and N. Itaco^{1,2}

¹*Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*

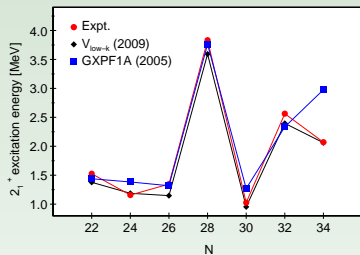
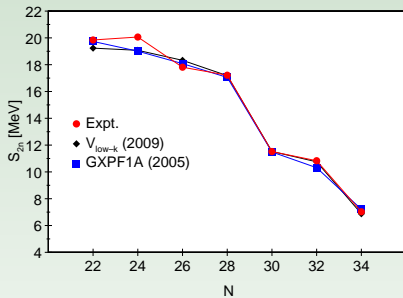
²*Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*

(Received 30 July 2009; published 12 October 2009)

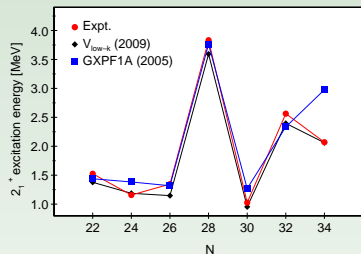
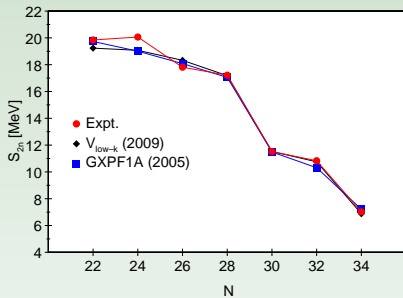
- ⇒ shell-model study of neutron-rich calcium isotopes
- ⇒ *fp* model space with ^{40}Ca inert core
- ⇒ predictions for the (at that time) unknown spectra of $^{53-56}\text{Ca}$



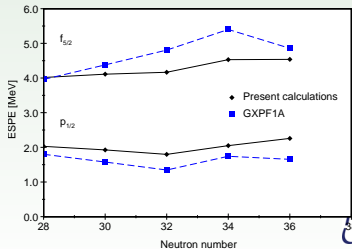
Heavy calcium isotopes: shell-model results



Heavy calcium isotopes: shell-model results



different monopole properties



Takeaway points

- predictive power of realistic shell model
- role of many-body correlations
- importance of HF insertions

