

Quantum Monte Carlo with chiral two- and three-neutron forces

Alexandros Gezerlis



ICNT Workshop: Theory for open-shell nuclei
East Lansing, MI
May 14, 2015

ICNT

International Collaborations in Nuclear Theory

2015 Approved and Supported Programs

International Collaborations in Nuclear Theory: Theory for open-shell nuclei near the limits of stability (May 11 - 29, 2015
MSU, East Lansing, MI) organized by S. K. Bogner, M. Hjorth-Jensen and J. D. Holt

2013 Approved and Supported Programs

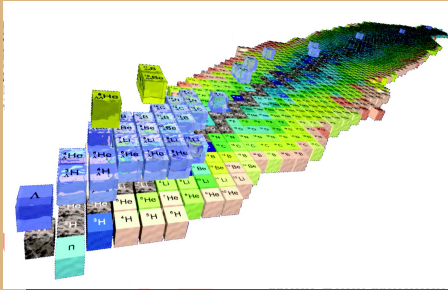
Halo Physics at the Neutron Dripline: Combining Ab Initio Nuclear Theory with Halo Effective Field Theory (GSI, Feb/Mar 2014)

ICNT workshop "Physics of exotic nuclei: Theoretical advances and challenges" (Tokyo, June 9-13, 2014)

2012 Approved and Supported Programs

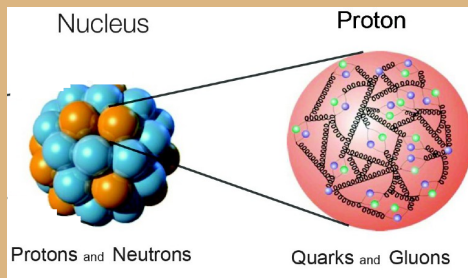
Symmetry Energy in the Context of New Radioactive Beam Facilities and Astrophysics (NSCL, July 15-August 9 2013)

Outline



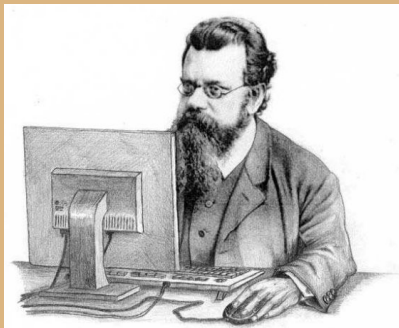
Many neutrons

- Neutron-rich nuclei
- Neutron stars



Nuclear forces

- Chiral Effective Field Theory
- Local chiral EFT



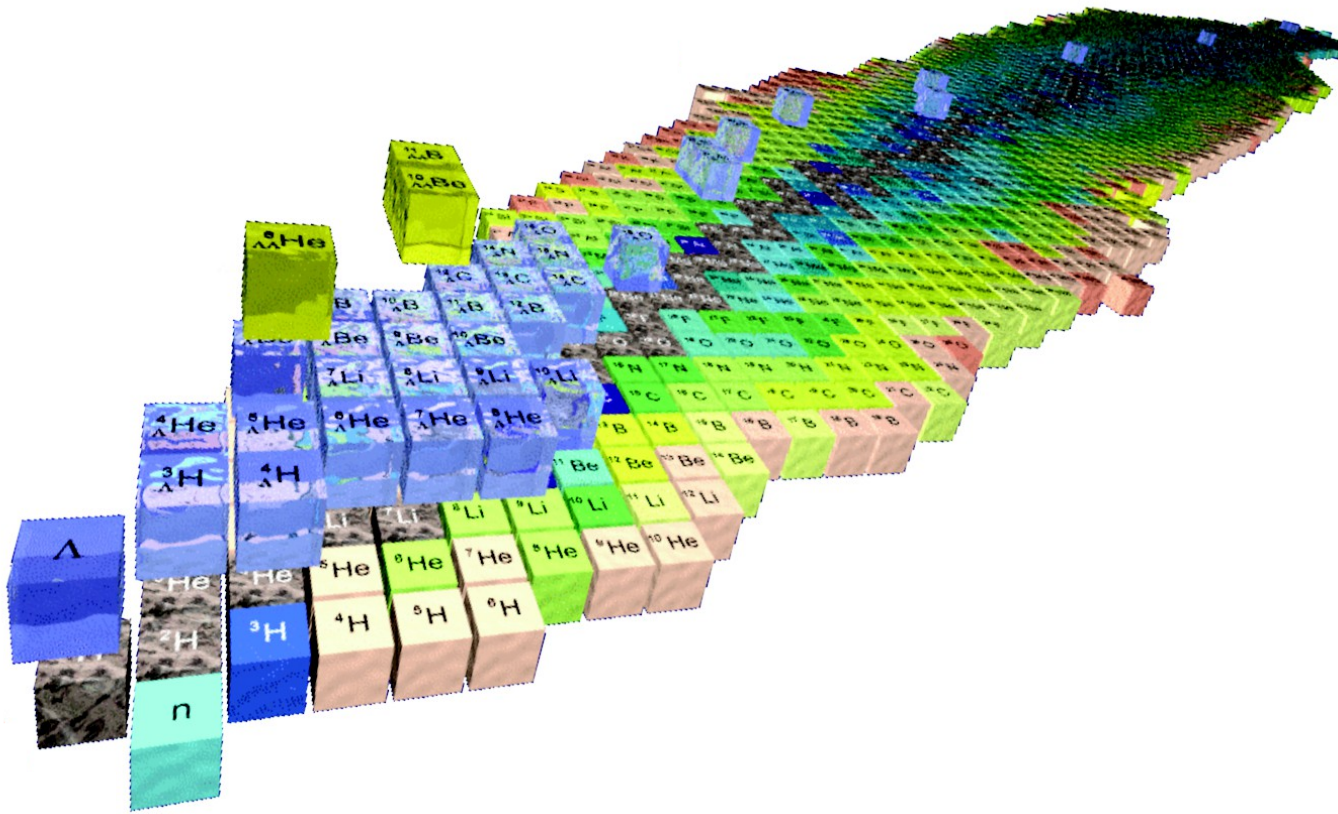
Credit: Bernhard Reischl

Results

- Neutron matter: Using NN forces alone
- Neutron matter: Using NN+3NF
- Neutron drops
- Neutron star crusts

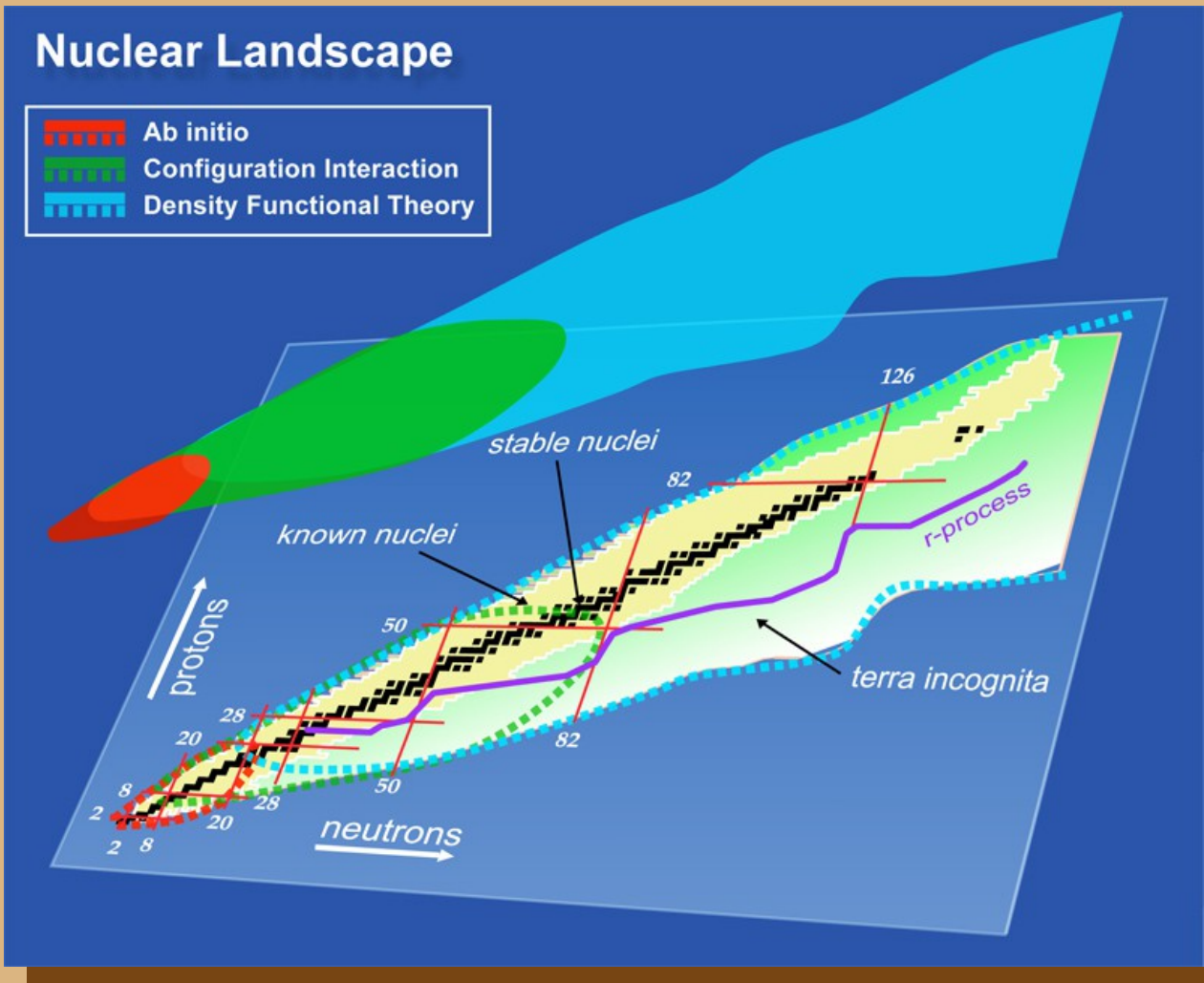
Many-nucleon problem: nuclei

Chart of nuclides



- Nuclear physics is developed and tested on earth
- Using complicated many-body methods we can try to “build nuclei from scratch”
- We then extrapolate that knowledge to more exotic systems

Many-nucleon problem: methods



- No universal method exists (yet?)
- A lot to be learned if the degrees of freedom are actual particles and there are no free parameters
- Regions of overlap between different methods are crucial
- Is it possible to work at the level of nucleons & pions but still connect to the underlying level?

Neutron-rich input

Quantum Monte Carlo:

stochastically solve the many-body Schrödinger equation in a fully non-perturbative manner

Energy-density functionals:

de facto include many-body correlations while being applicable throughout the nuclear chart

Rudiments of Diffusion Monte Carlo:

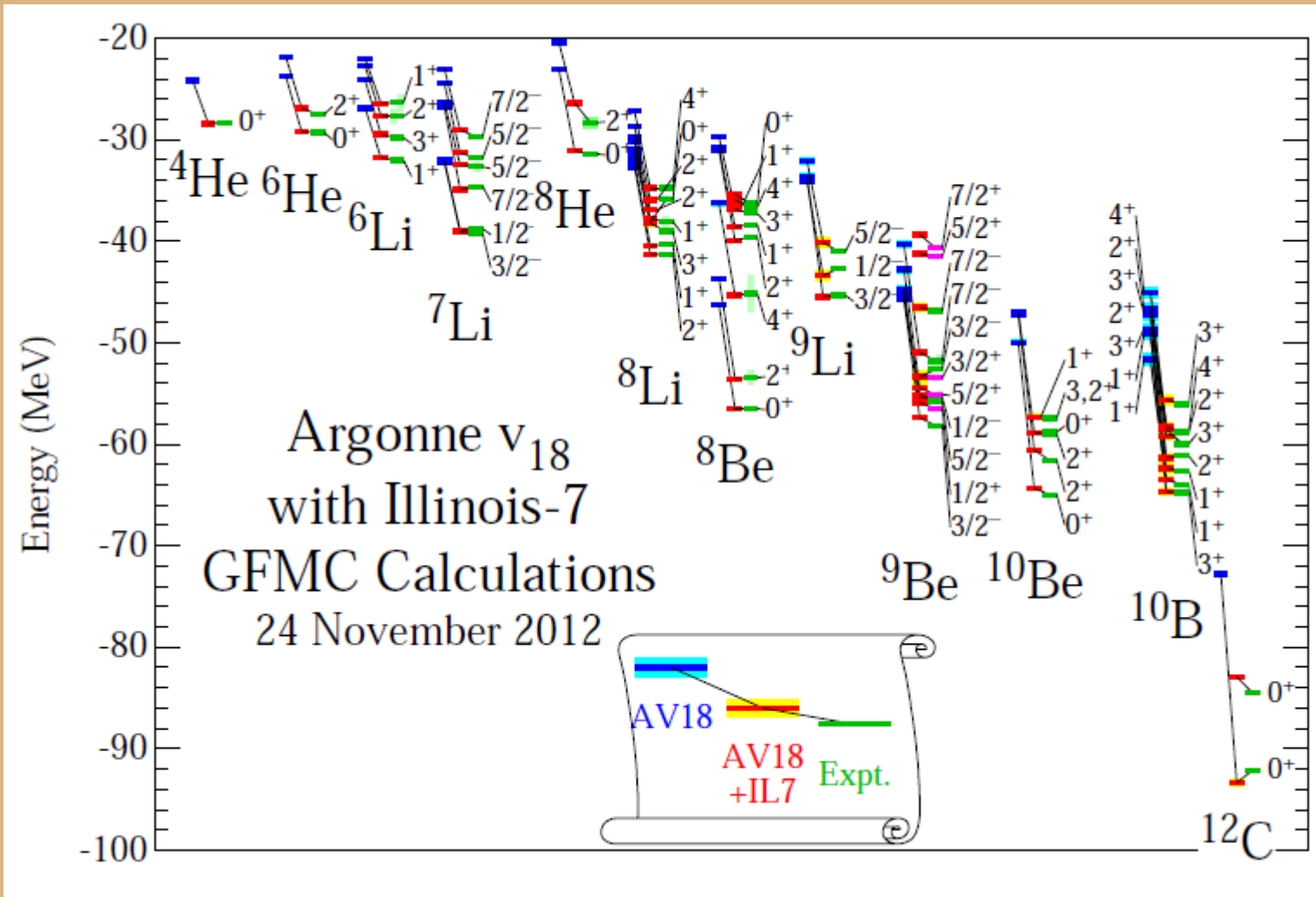
$$\begin{aligned}\Psi(\tau \rightarrow \infty) &= \lim_{\tau \rightarrow \infty} e^{-(\mathcal{H}-E_T)\tau} \Psi_V \\ &\rightarrow \alpha_0 e^{-(E_0-E_T)\tau} \Psi_0\end{aligned}$$

Rudiments of Skyrme EDFs:

$$\begin{aligned}\mathcal{E}_{\text{Skyrme}} &= \frac{\hbar^2}{2m} \tau + \frac{s_0}{4} \rho^2 + \frac{s_3}{24} \rho^{\alpha+2} \\ &\quad + \frac{s_1 + 3s_2}{8} \rho \tau \\ &\quad + 3 \frac{s_1 - s_2}{16} (\nabla \rho)^2\end{aligned}$$

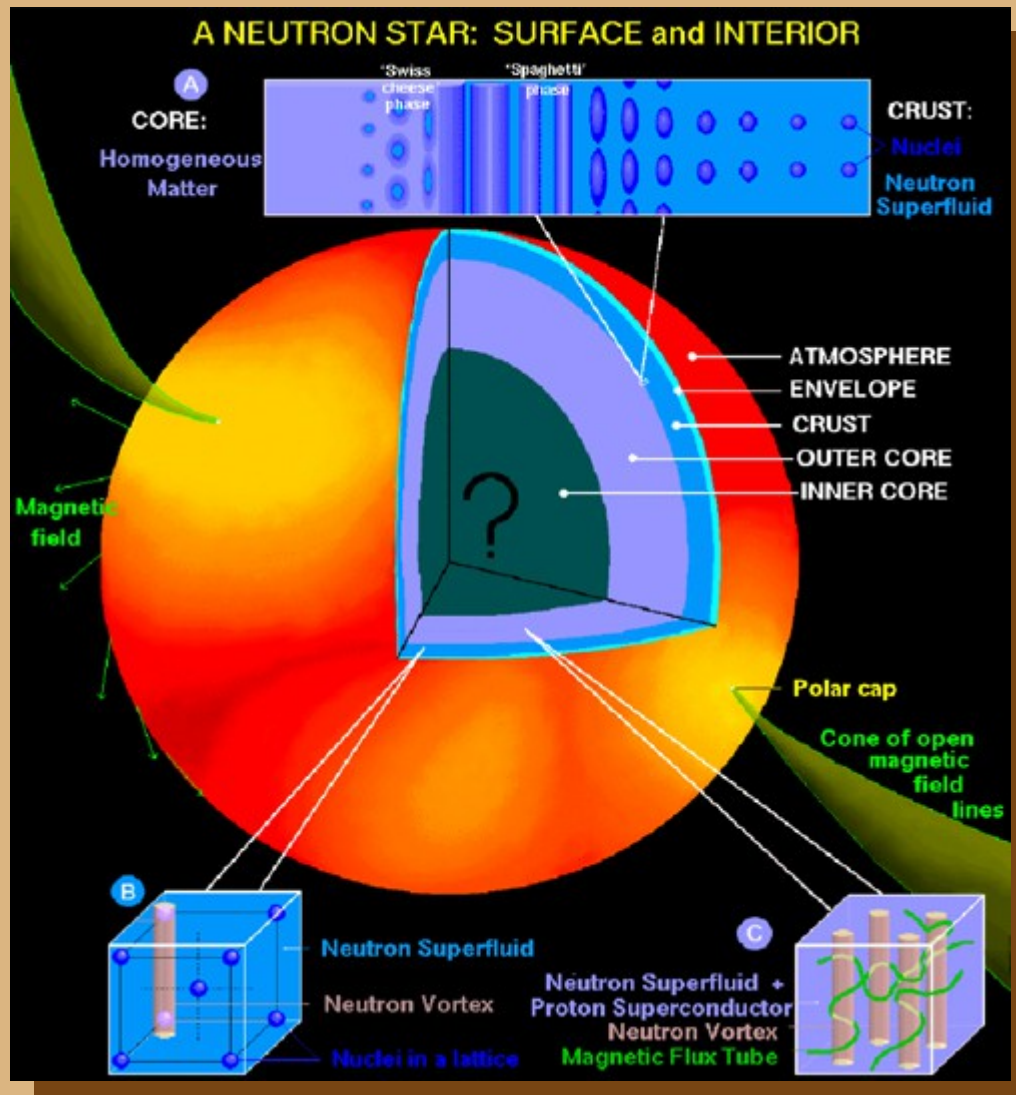
Nuclear GFMC preview

Joel Lynn's talk: nuclear Green's Function Monte Carlo is very accurate



Neutron stars

Ultra-dense matter laboratories



- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, but also (indirectly) experimentally accessible
- We wish to describe neutron-star matter from first principles

Neutron stars: gravity

Static spherically symmetric metric

$$g_{\mu\nu} = \begin{pmatrix} A(r) & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2 \sin^2 \theta & 0 \\ 0 & 0 & 0 & -B(r) \end{pmatrix}$$

Einstein field equations

$$R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right)$$

Ricci tensor

Energy-momentum tensor

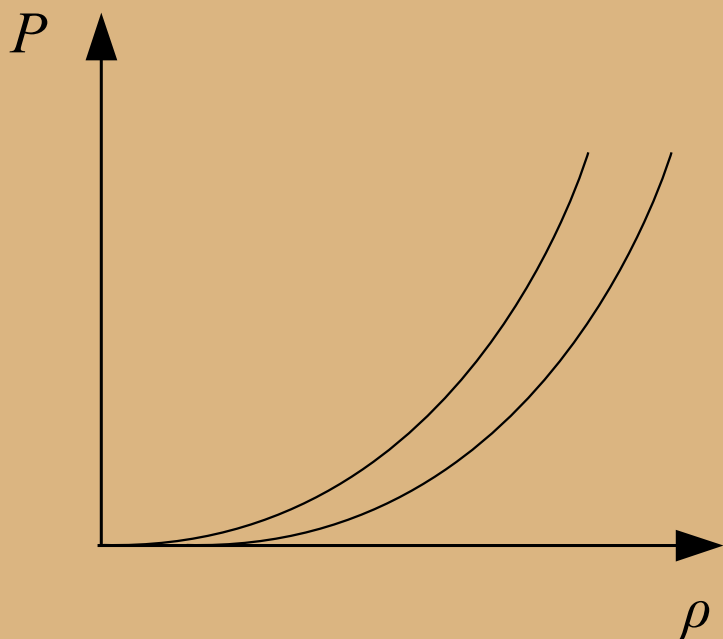
Inside the source (for isotropic fluid with no shear forces):
Tolman-Oppenheimer-Volkoff (TOV) equation(s)

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \left[\frac{1 + P(r)/\rho(r)c^2}{1 - 2GM(r)/c^2 r} \right] \left[1 + \frac{4\pi P(r)r^3}{M(r)c^2} \right]$$

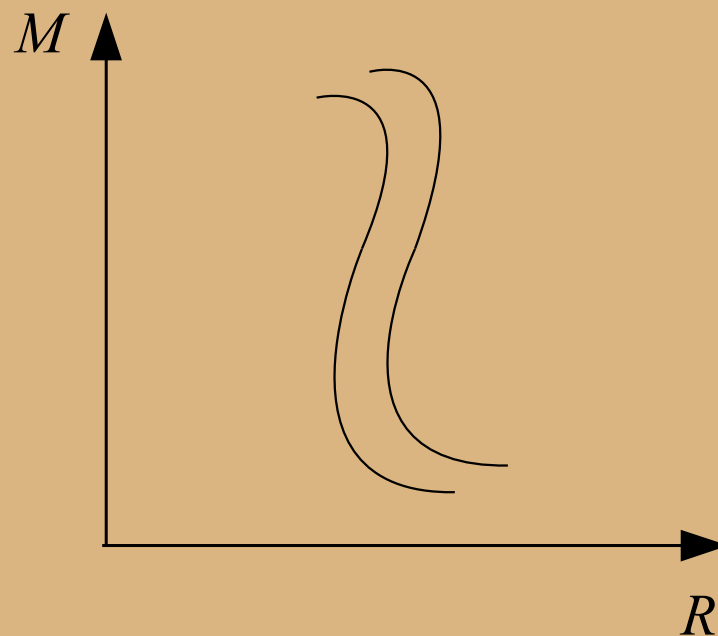
Uncertainty estimates

TOV equations
(or Hartle-Thorne, etc)

Pressure vs density

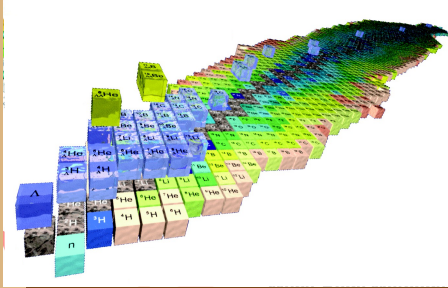


Mass vs radius



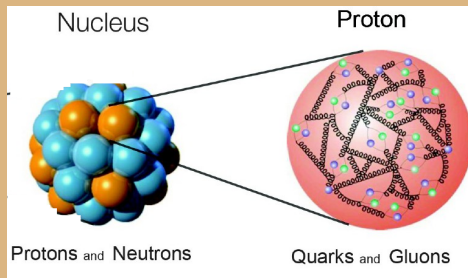
Modern goal: systematic theoretical error bars

Outline



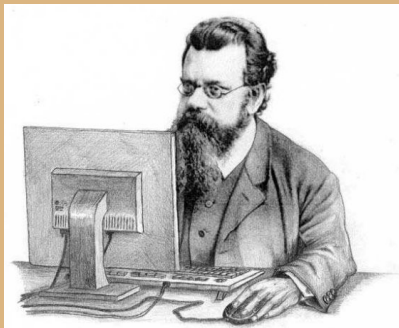
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Nuclear forces

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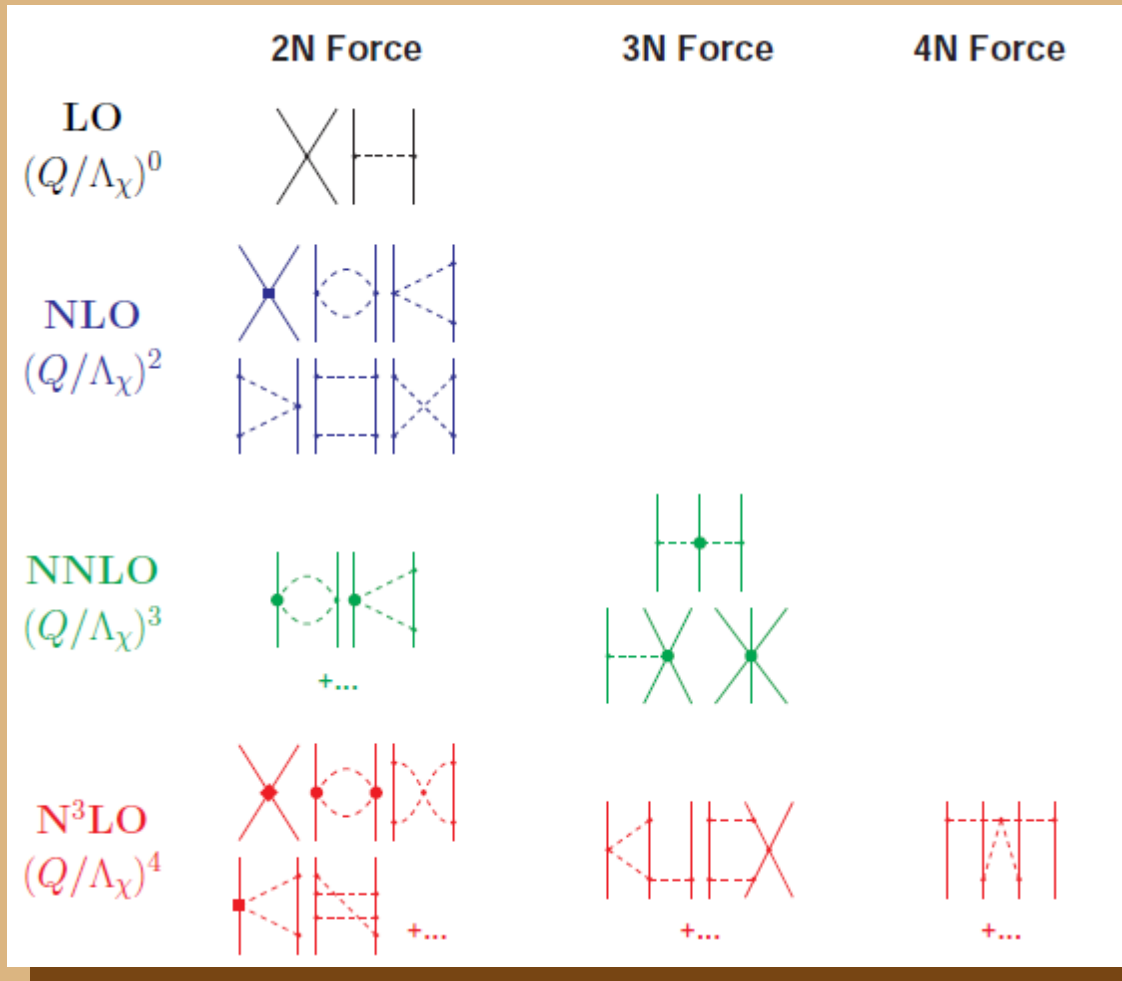


Credit: Bernhard Reischl

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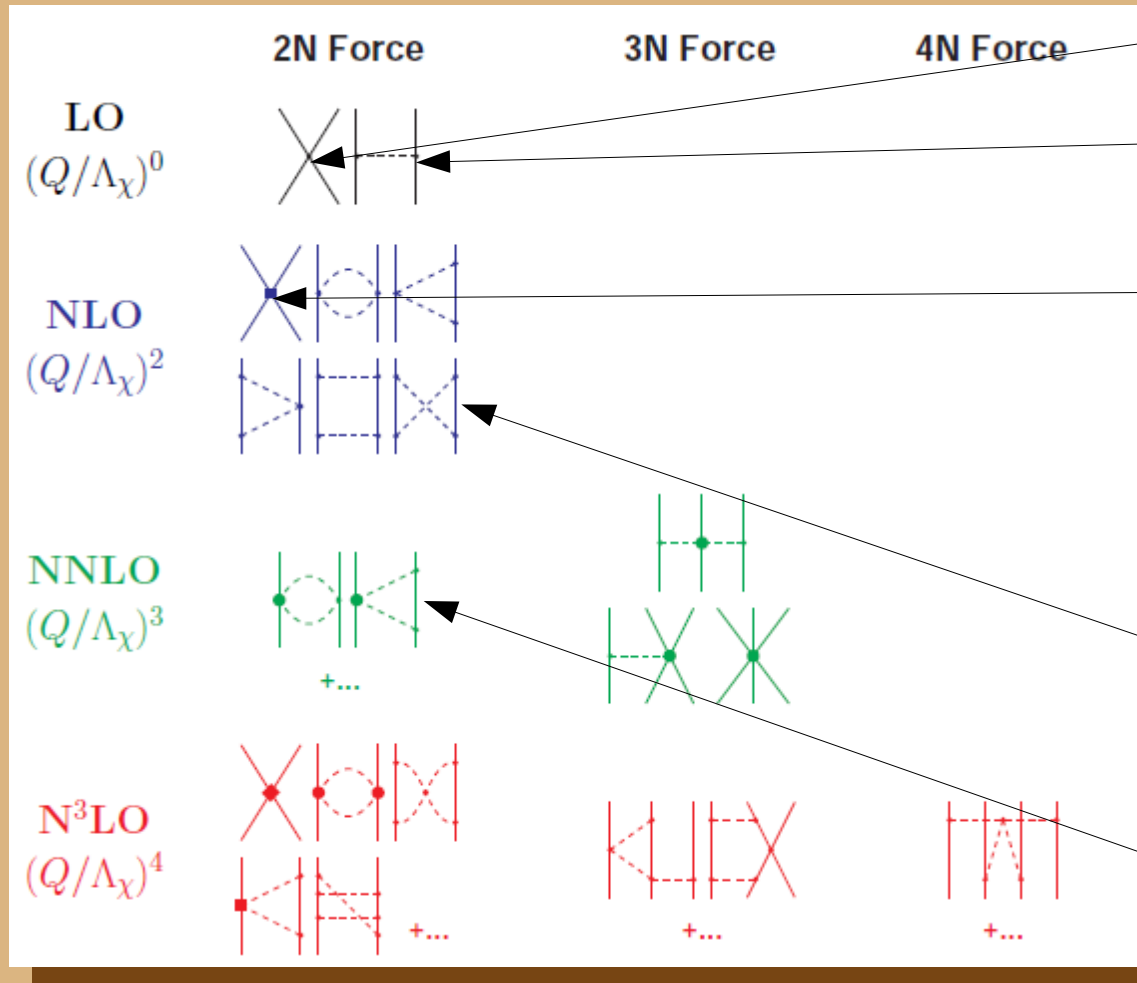
Nuclear Hamiltonian: chiral EFT



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until recently non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization still an open question

Weinberg, van Kolck, Kaplan, Savage, Wise, Machleidt, Epelbaum, ...

Nuclear Hamiltonian: chiral EFT



$$V_{\text{ct}}^{(0)} = C_S + C_T \sigma_1 \cdot \sigma_2$$

$$V_{1\pi}^{(0)} = - \left(\frac{g_A}{2f_\pi} \right)^2 \tau_1 \cdot \tau_2 \frac{(\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})}{q^2 + m_\pi^2}$$

$$V_{\text{ct}}^{(2)} = C_1 q^2 + C_2 k^2 + (C_3 q^2 + C_4 k^2) \sigma_1 \cdot \sigma_2 + i \frac{C_5}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{q} \times \mathbf{k}) + C_6 (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q}) + C_7 (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})$$

Long-studied two-pion exchange

Contains couplings from πN scattering

$$\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$$

$$\mathbf{k} = (\mathbf{p}' + \mathbf{p})/2$$

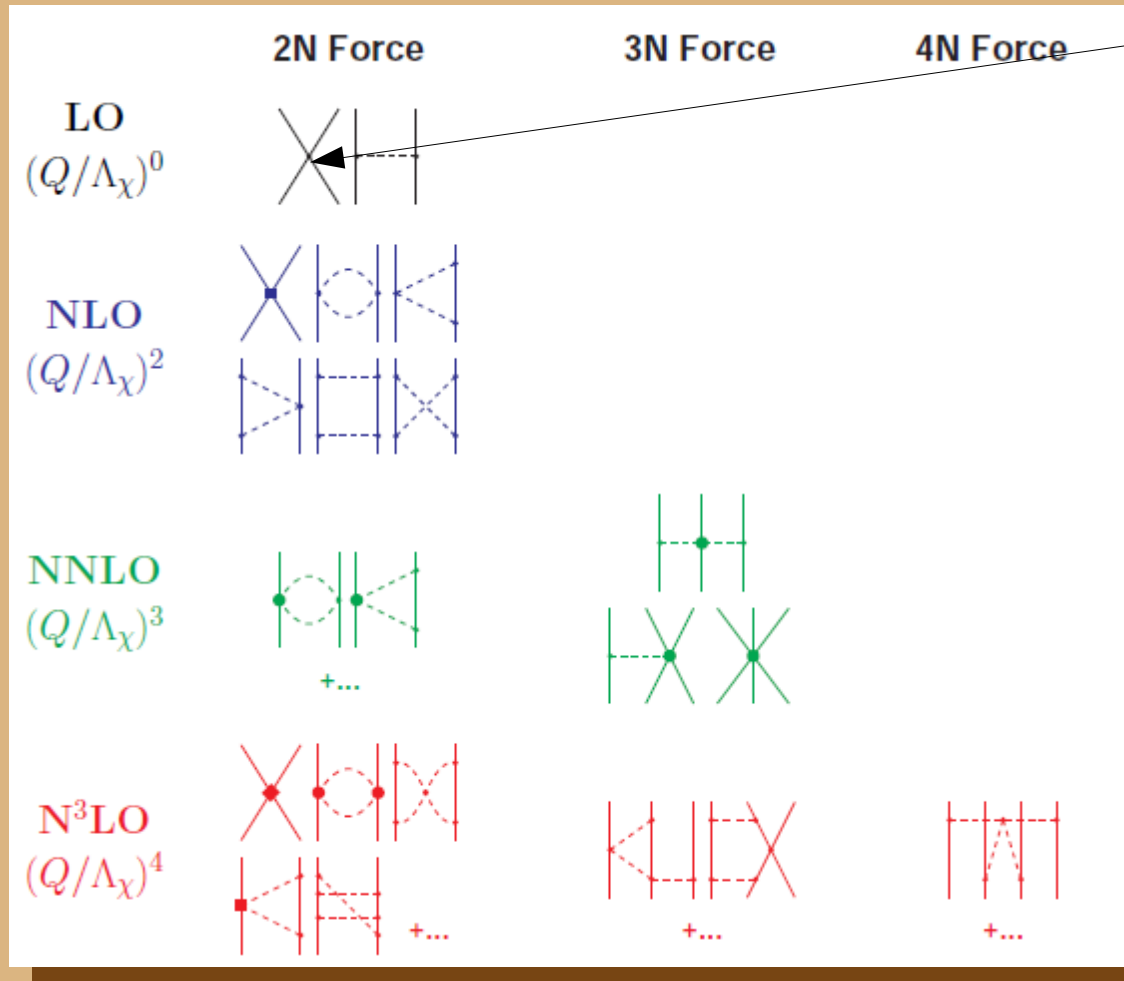
$$\mathbf{p}' = (\mathbf{p}'_1 - \mathbf{p}'_2)/2$$

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}$$

\mathbf{k} means non-local

\mathbf{q} means local

Nuclear Hamiltonian: chiral EFT



$$V_{\text{ct}}^{(0)} = C_S + C_T \sigma_1 \cdot \sigma_2$$

Merely the standard choice.

Actually 4 terms in full set
consistent with the symmetries of QCD

$$V_{\text{ct}}^{(0)} = C_1 + C_2 \sigma_1 \cdot \sigma_2 + C_3 \tau_1 \cdot \tau_2 + C_4 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$

Pick 2 and antisymmetrize

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C **90**, 054323 (2014).

Local chiral EFT

Use the analogous freedom for NLO contacts

Write down a local energy-independent NN potential

- Pick 7 different contacts at NLO, just make sure that when antisymmetrized they lead to a set obeying the required symmetry principles

$$\begin{aligned} V_{\text{ct}}^{(2)} = & C_1 q^2 + C_2 q^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\ & + (C_3 q^2 + C_4 q^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + i \frac{C_5}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{q} \times \mathbf{k} \\ & + C_6 (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\ & + C_7 (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \end{aligned}$$

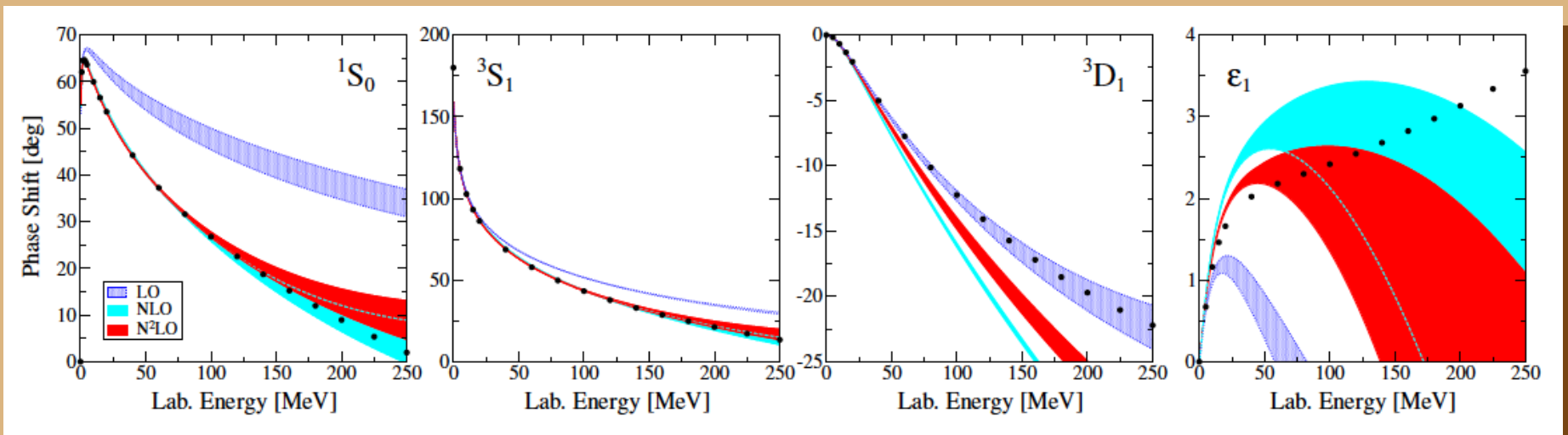
cf.

$$\begin{aligned} V_{\text{ct}}^{(2)} = & C_1 q^2 + C_2 k^2 \\ & + (C_3 q^2 + C_4 k^2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + i \frac{C_5}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) \\ & + C_6 (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\ & + C_7 (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \end{aligned}$$

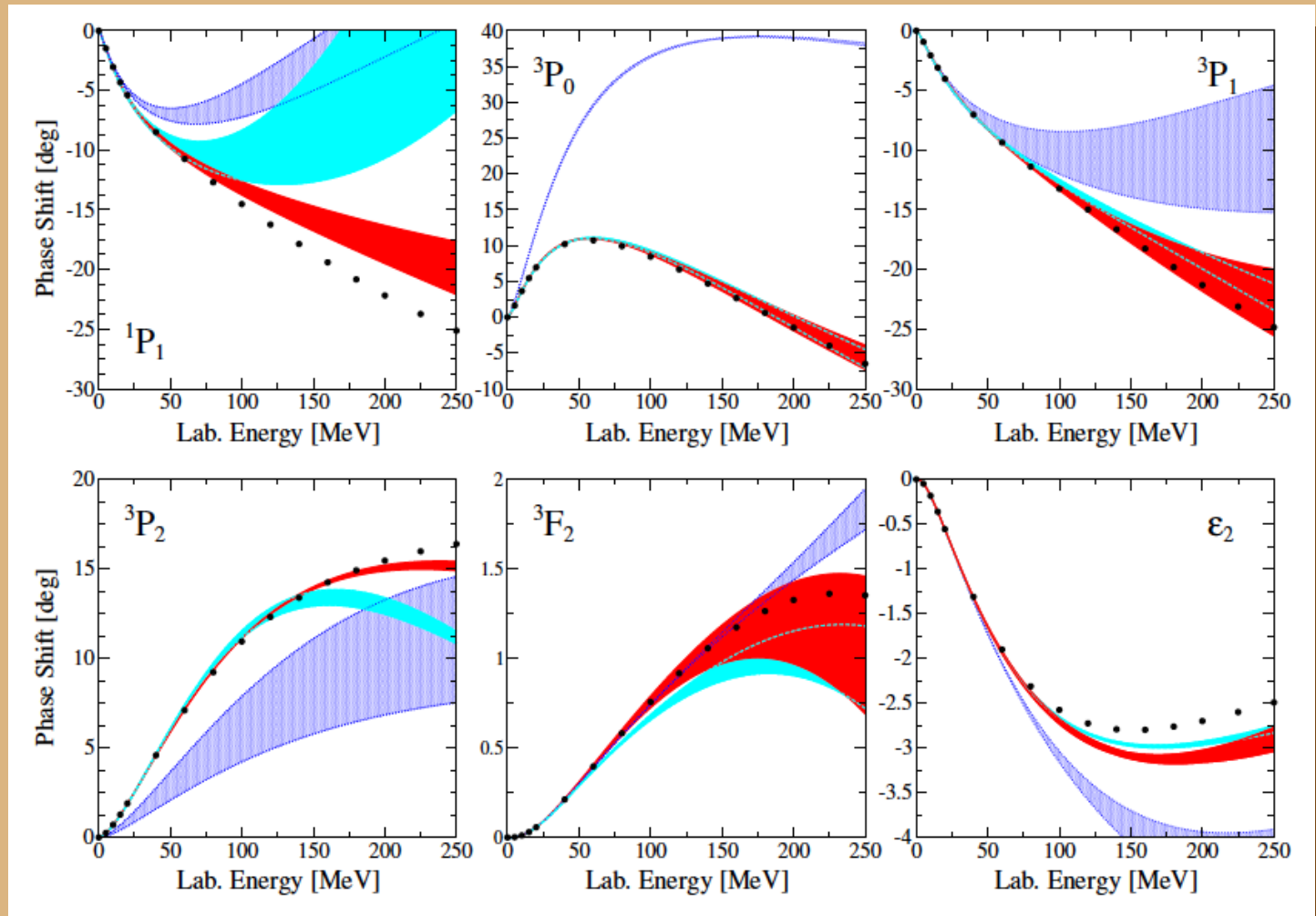
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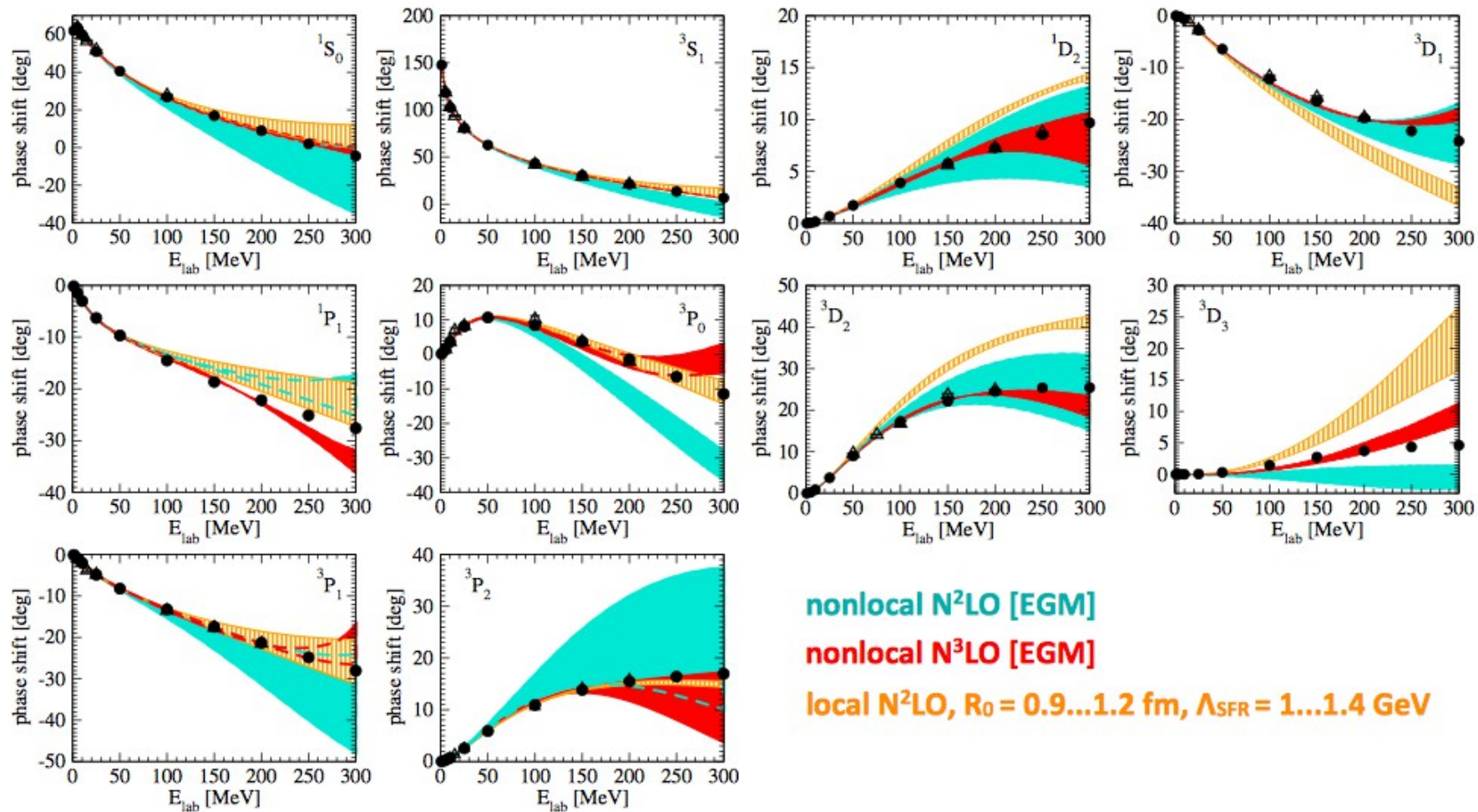
Phase shifts



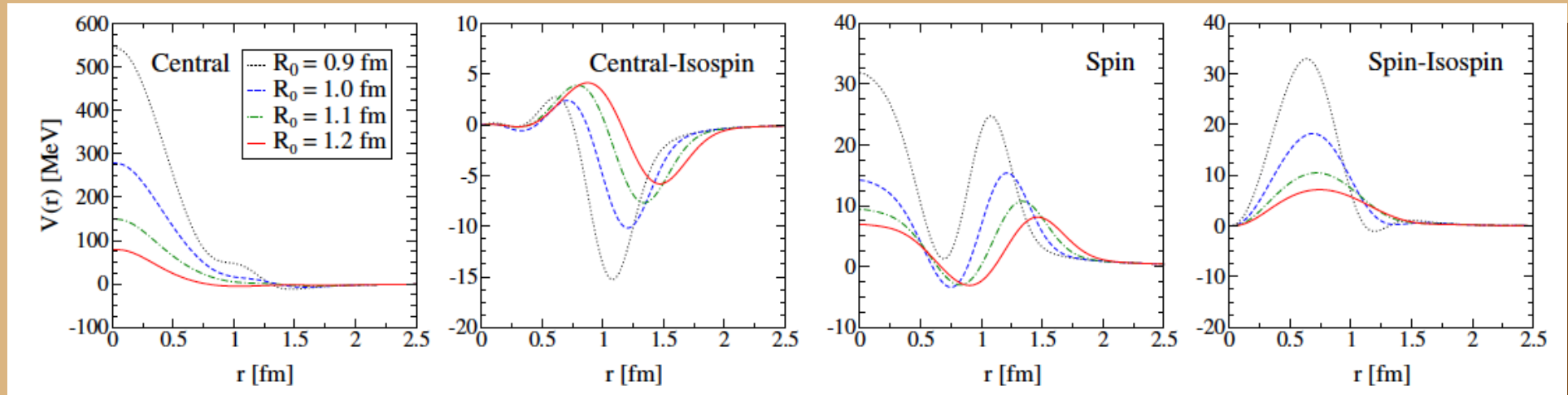
Phase shifts



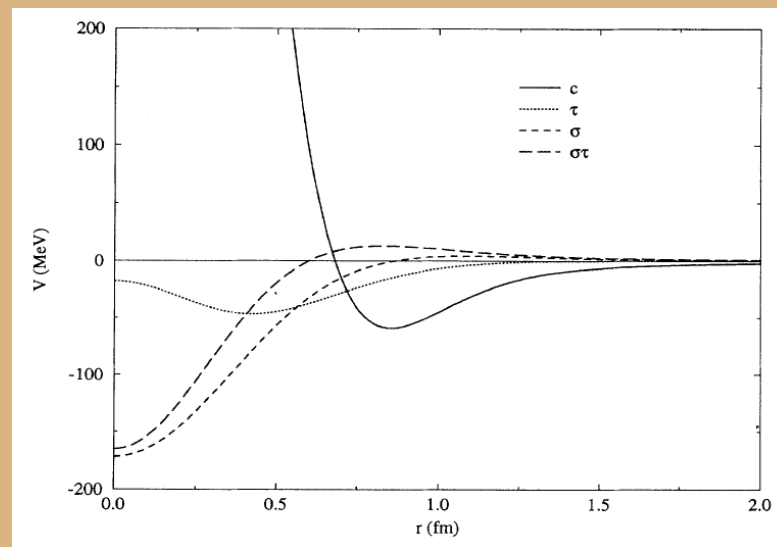
Compare to non-local EGM



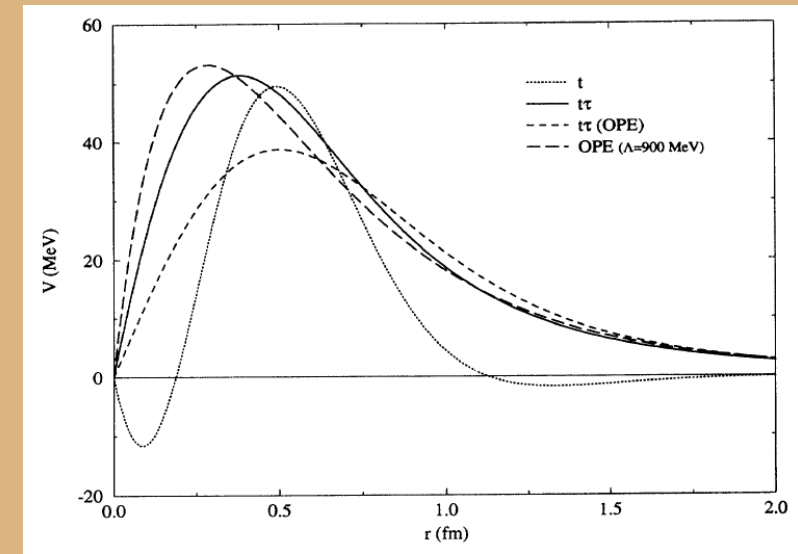
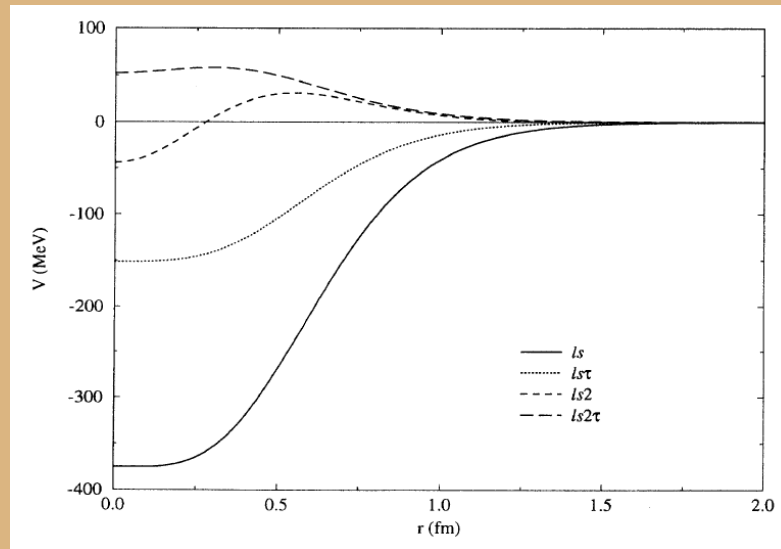
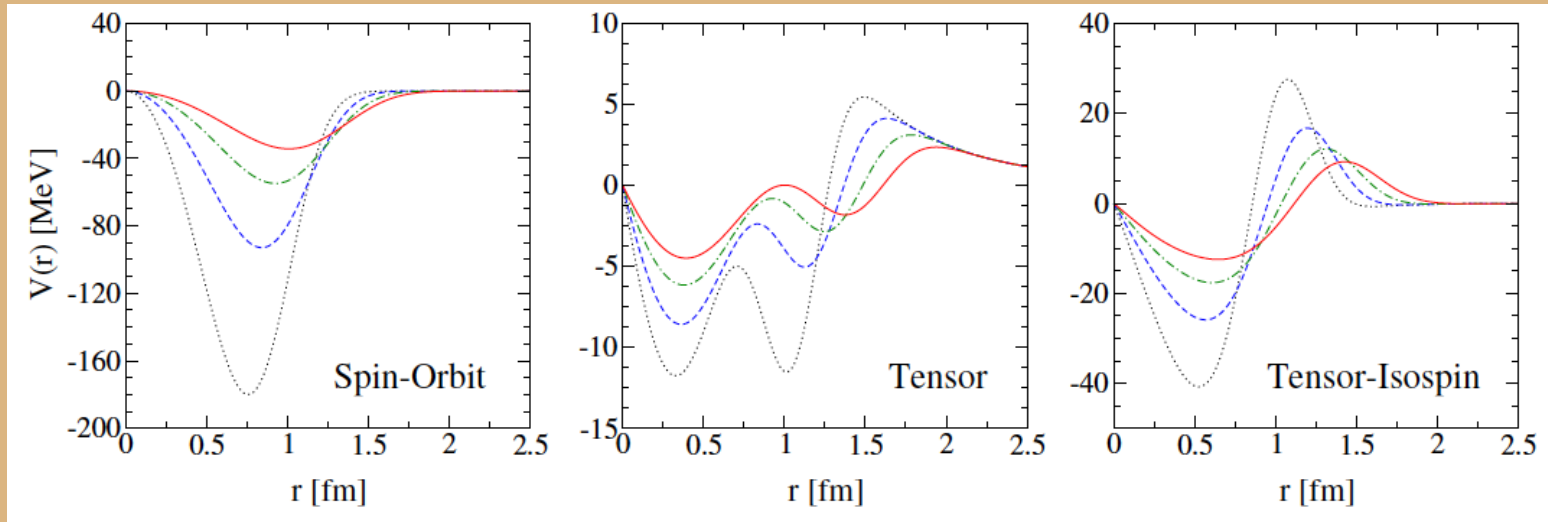
Since it's local, let's plot it (N²LO)



Compare
with
AV18

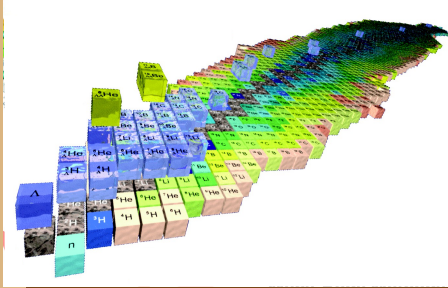


Since it's local, let's plot it (N²LO)



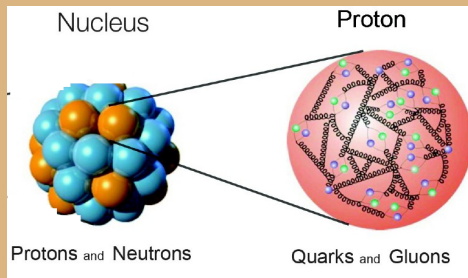
Compare
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Nuclear forces

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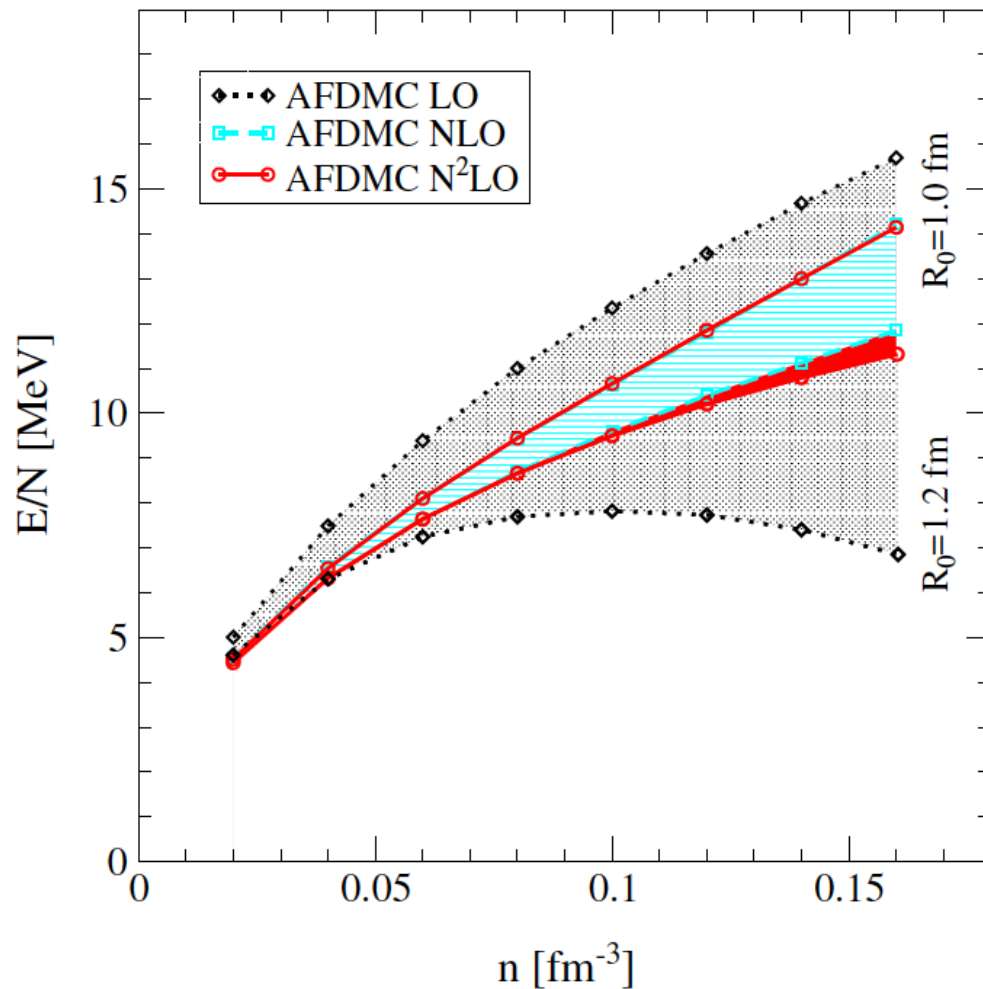


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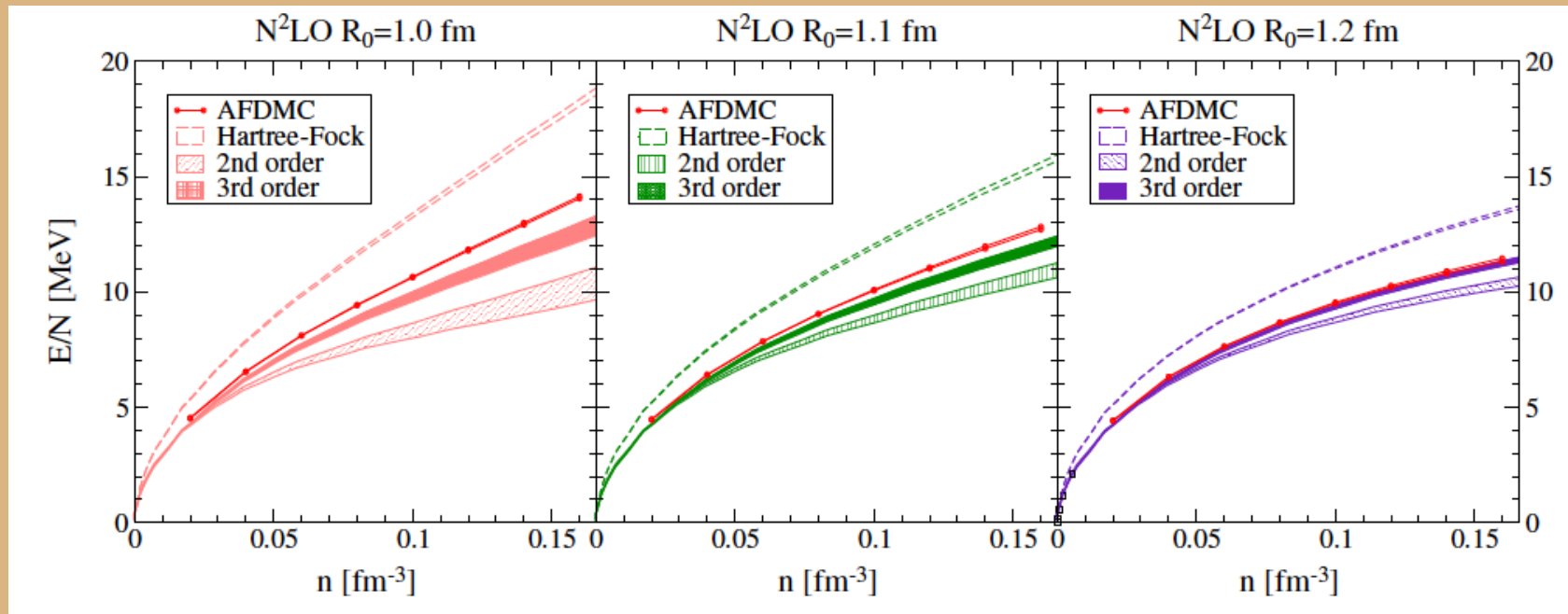
Chiral EFT in QMC



- Use Auxiliary-Field Diffusion Monte Carlo to handle the full interaction
- First ever non-perturbative systematic error bands
- Band sizes to be expected
- Many-body forces will emerge systematically

NEUTRONS

QMC vs MBPT



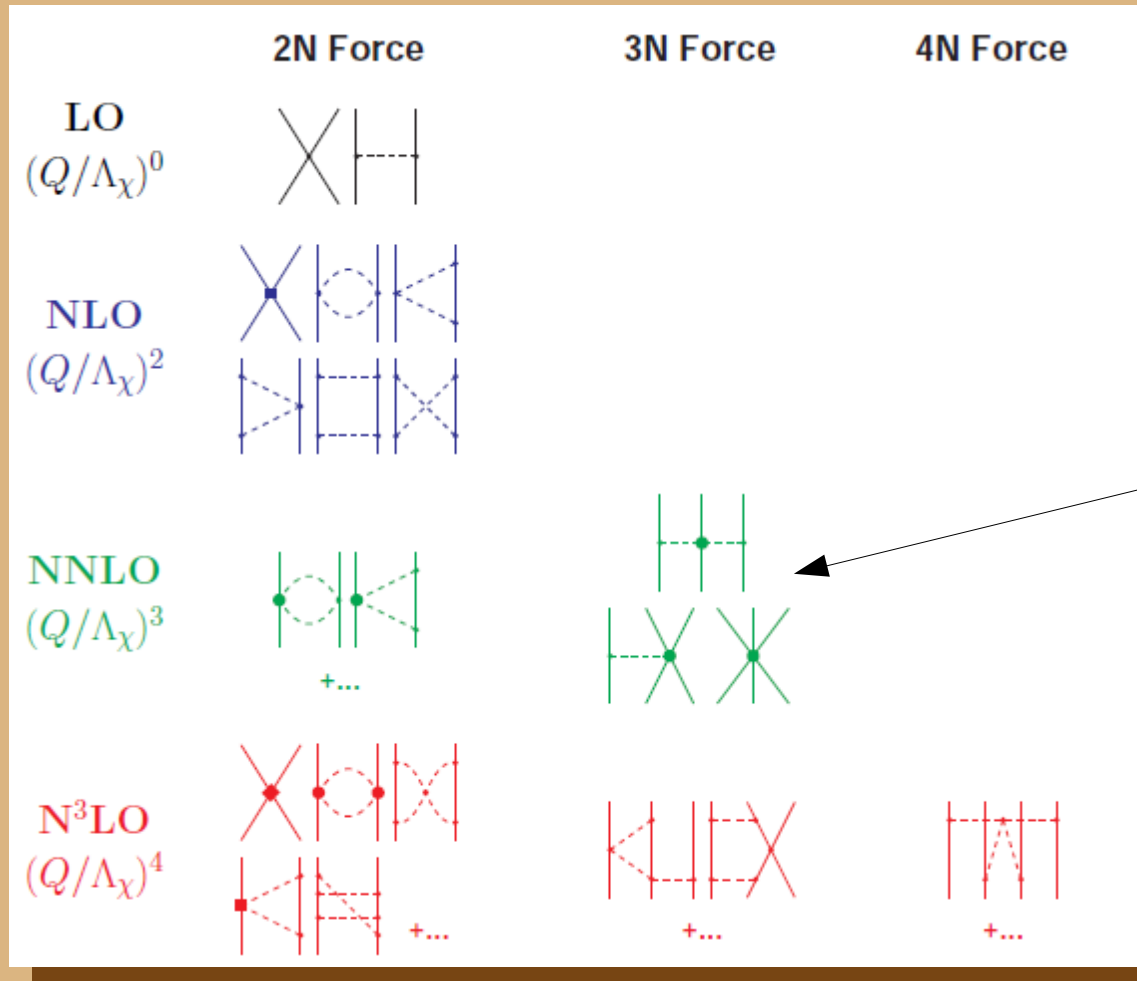
- MBPT bands come from diff. single-particle spectra
- Soft potential in excellent agreement with AFDMC

NEUTRONS

What about three-nucleon forces?

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, *in preparation*

Nuclear Hamiltonian: chiral EFT



N2LO 3NF

- Two-pion exchange (parameter-free)
- One-pion exchange-contact (c_D)
- Three-nucleon contact (c_E)

V_D and V_E are merely regulator effects in PNM

3NF TPE in PNM

Momentum space

$$V_{\text{TPE}}^{\text{PNM}} = \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_k \cdot \mathbf{q}_k)}{(q_i^2 + m_\pi^2)(q_k^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q}_i \cdot \mathbf{q}_k \right]$$

Coordinate space

$$\begin{aligned} V_{\text{TPE}}^{\text{PNM}} = & -\frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{m_\pi}{4\pi} \right)^2 \left(-\frac{4c_1 m_\pi^2}{f_\pi^2} \right) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kj} U(r_{ij}) Y(r_{ij}) U(r_{kj}) Y(r_{kj}) \\ & + \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{1}{4\pi} \right)^2 \left(\frac{2c_3}{f_\pi^2} \right) \left[\frac{m_\pi^4}{9} X_{ij}(\mathbf{r}_{ij}) X_{kj}(\mathbf{r}_{kj}) - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right. \\ & \left. - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{kj}) \delta(\mathbf{r}_{ij}) + \frac{(4\pi)^2}{9} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k \delta(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right] \end{aligned}$$

3NF TPE in PNM

Momentum space

$$V_{\text{TPE}}^{\text{PNM}} = \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_k \cdot \mathbf{q}_k)}{(q_i^2 + m_\pi^2)(q_k^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q}_i \cdot \mathbf{q}_k \right]$$

Coordinate space

$$V_{\text{TPE}}^{\text{PNM}} = -\frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{m_\pi}{4\pi} \right)^2 \left(-\frac{4c_1 m_\pi^2}{f_\pi^2} \right) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kj} U(r_{ij}) Y(r_{ij}) U(r_{kj}) Y(r_{kj})$$

$$+ \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{1}{4\pi} \right)^2 \left(\frac{2c_3}{f_\pi^2} \right) \left[\frac{m_\pi^4}{9} X_{ij}(\mathbf{r}_{ij}) X_{kj}(\mathbf{r}_{kj}) - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right.$$

$$\left. - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{kj}) \delta(\mathbf{r}_{ij}) + \frac{(4\pi)^2}{9} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k \delta(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right]$$

Long-range (LR)

3NF TPE in PNM

Momentum space

$$V_{\text{TPE}}^{\text{PNM}} = \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_k \cdot \mathbf{q}_k)}{(q_i^2 + m_\pi^2)(q_k^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q}_i \cdot \mathbf{q}_k \right]$$

Coordinate space

$$V_{\text{TPE}}^{\text{PNM}} = -\frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{m_\pi}{4\pi} \right)^2 \left(-\frac{4c_1 m_\pi^2}{f_\pi^2} \right) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kj} U(r_{ij}) Y(r_{ij}) U(r_{kj}) Y(r_{kj})$$

$$+ \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{1}{4\pi} \right)^2 \left(\frac{2c_3}{f_\pi^2} \right) \left[\frac{m_\pi^4}{9} X_{ij}(\mathbf{r}_{ij}) X_{kj}(\mathbf{r}_{kj}) - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right.$$

$$\left. - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{kj}) \delta(\mathbf{r}_{ij}) + \frac{(4\pi)^2}{9} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k \delta(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right]$$

Intermediate-range (IR)


3NF TPE in PNM

Momentum space

$$V_{\text{TPE}}^{\text{PNM}} = \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_k \cdot \mathbf{q}_k)}{(q_i^2 + m_\pi^2)(q_k^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q}_i \cdot \mathbf{q}_k \right]$$

Coordinate space

$$V_{\text{TPE}}^{\text{PNM}} = -\frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{m_\pi}{4\pi} \right)^2 \left(-\frac{4c_1 m_\pi^2}{f_\pi^2} \right) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kj} U(r_{ij}) Y(r_{ij}) U(r_{kj}) Y(r_{kj})$$

Short-range (SR) 

$$+ \frac{1}{2} \left(\frac{g_A}{2f_\pi} \right)^2 \left(\frac{1}{4\pi} \right)^2 \left(\frac{2c_3}{f_\pi^2} \right) \left[\frac{m_\pi^4}{9} X_{ij}(\mathbf{r}_{ij}) X_{kj}(\mathbf{r}_{kj}) - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right.$$
$$\left. - \frac{4\pi m_\pi^2}{9} X_{ik}(\mathbf{r}_{kj}) \delta(\mathbf{r}_{ij}) + \frac{(4\pi)^2}{9} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k \delta(\mathbf{r}_{ij}) \delta(\mathbf{r}_{kj}) \right]$$

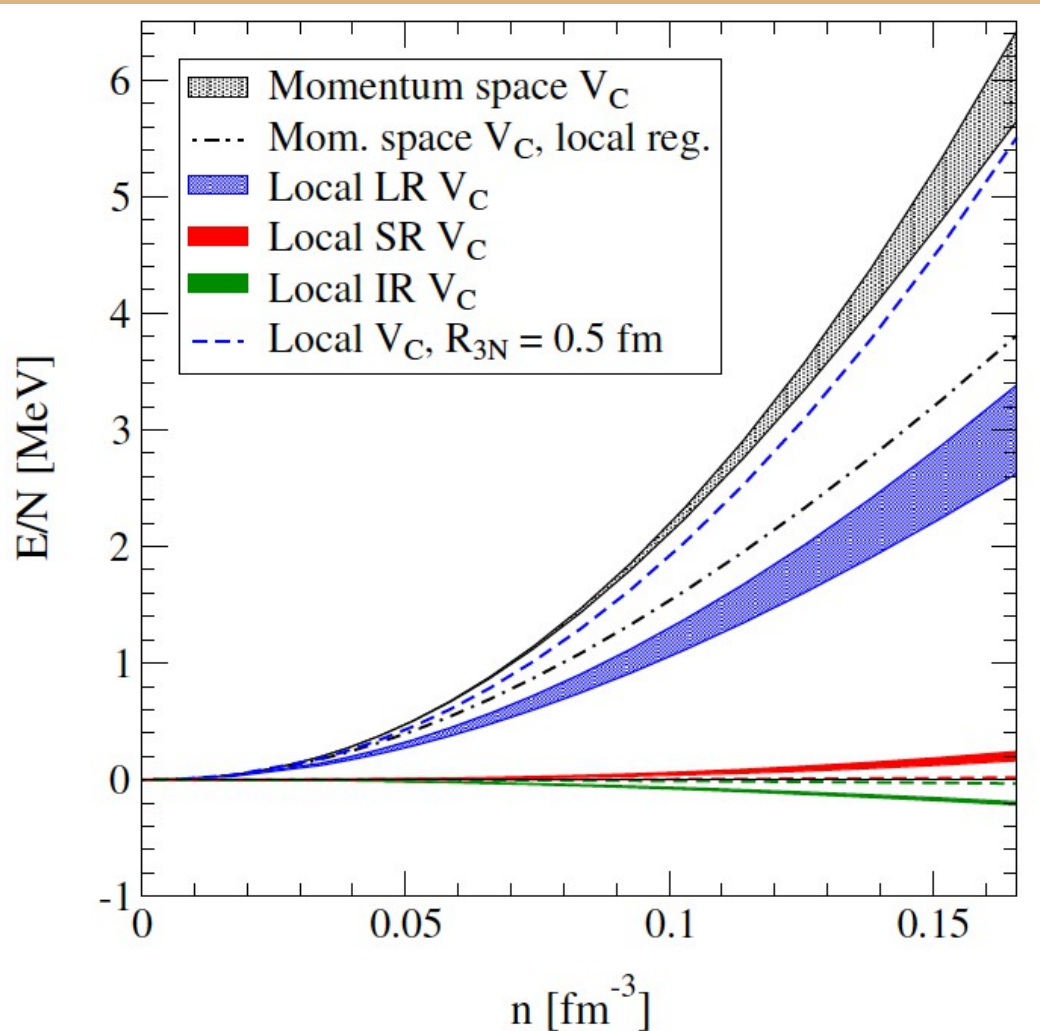
Regularizing

Attempt to be consistent with NN regularization

$$\delta(\mathbf{r}) \rightarrow \delta_{R_{3N}}(\mathbf{r}) = \frac{1}{\pi\Gamma(3/4)R_{3N}^3} e^{-(r/R_{3N})^4}$$

$$Y(r) \rightarrow Y(r) \left(1 - e^{-(r/R_{3N})^4}\right)$$

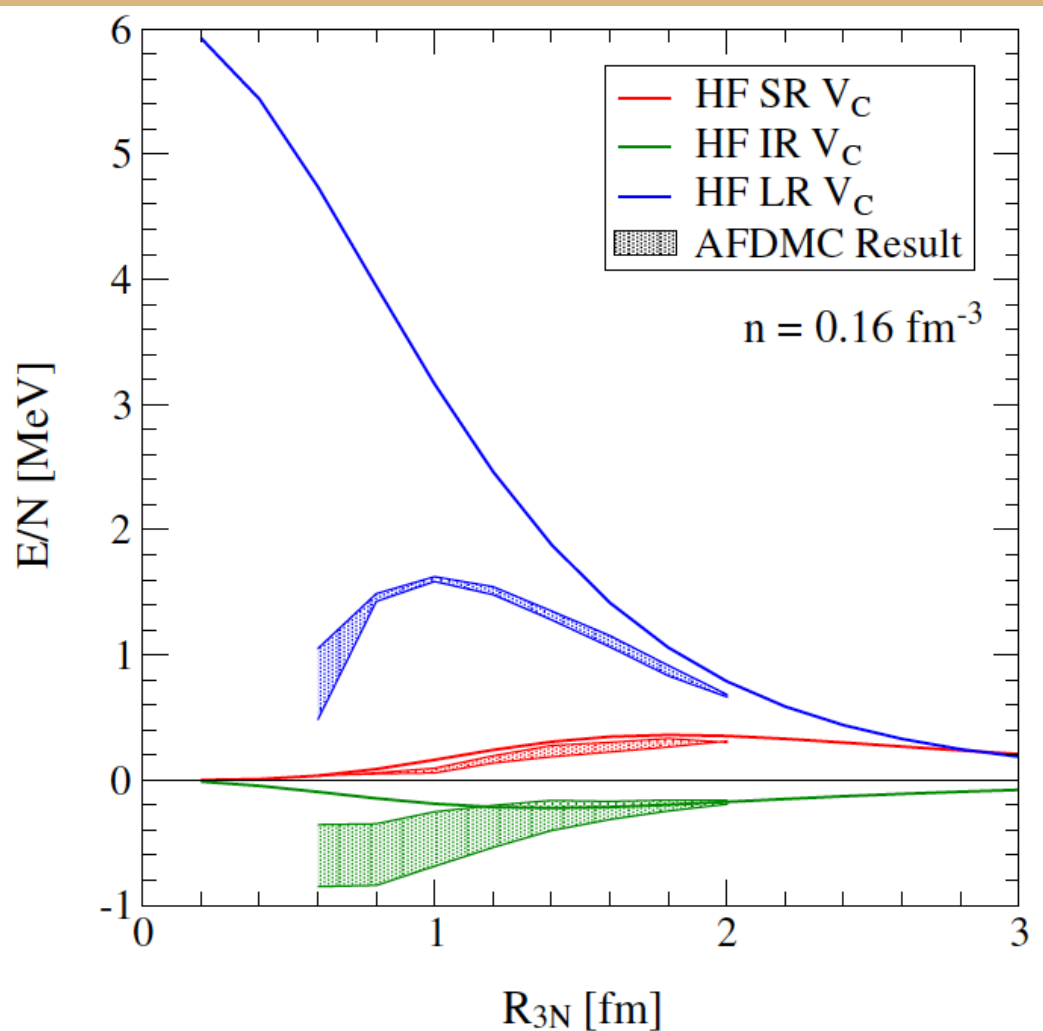
3NF contributions: Hartree-Fock



- Local 3NFs (Navratil + ours) lead to ~ 2.5 MeV lower energy at saturation density
- Our new local 3NF results agree with Navratil local 3NF result for very short coordinate-space cutoff

NEUTRONS

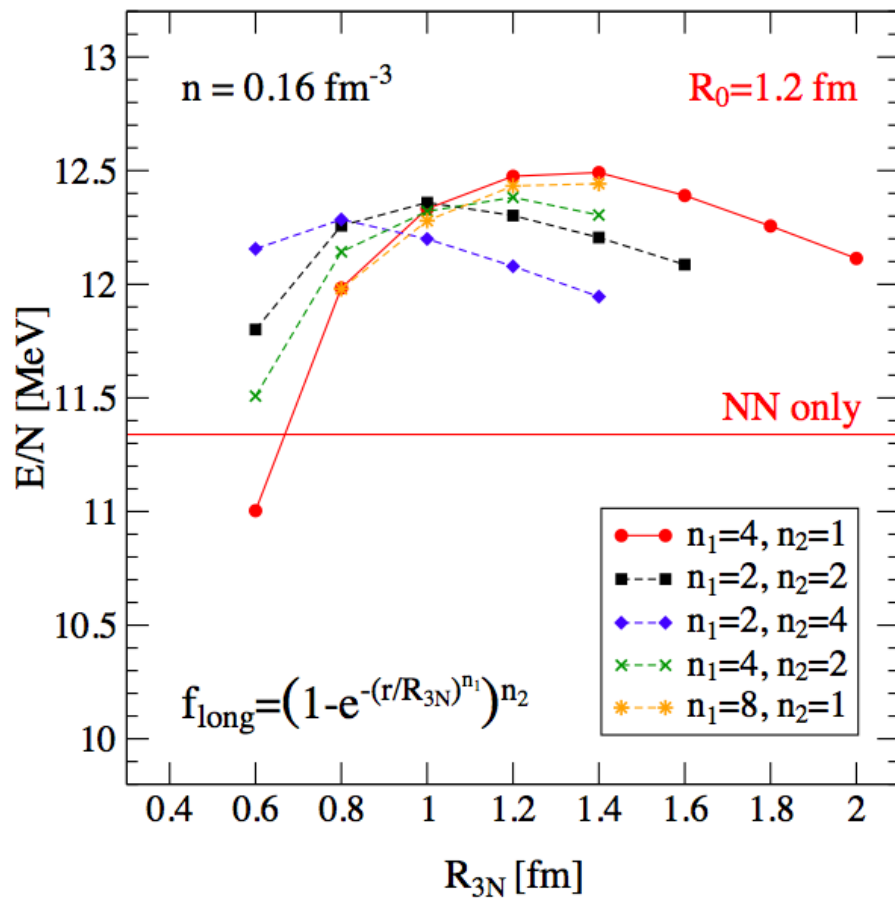
3NF contributions



- Shown are both HF (lines) and AFDMC (bands) for 3NF contrib
- HF shows IR & SR vanishing at low R_{3N}
- AFDMC at low R_{3N} shows collapse of LR and IR

NEUTRONS

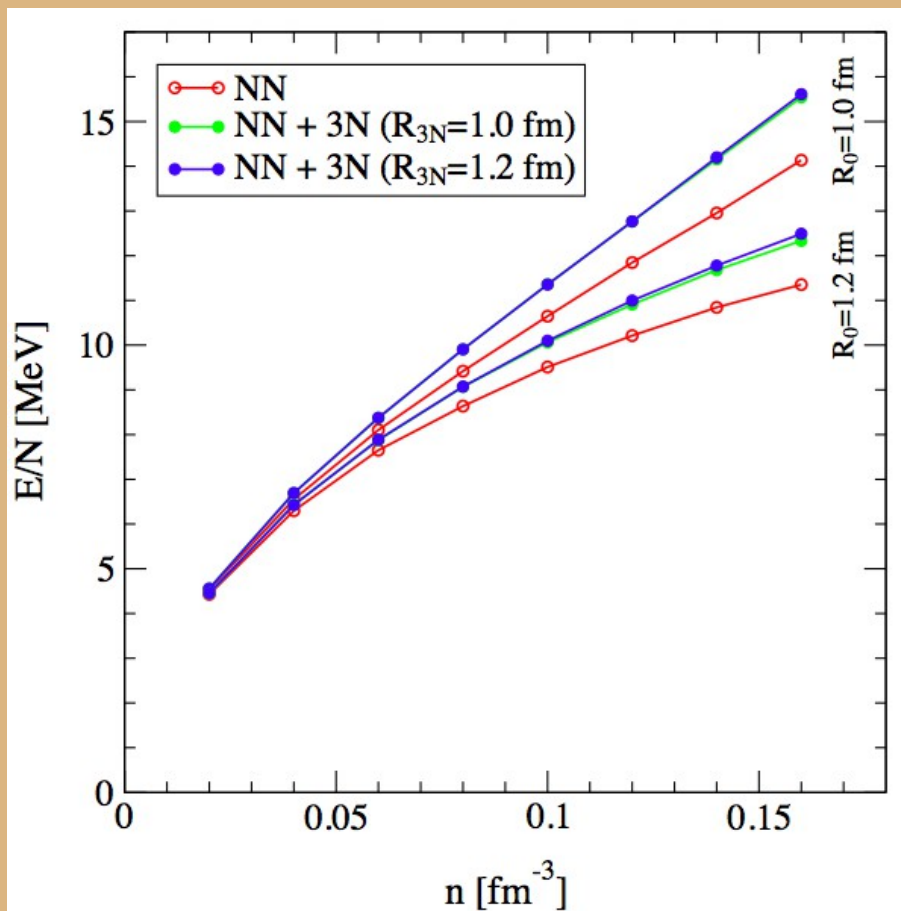
3NF cutoff choice



- Too large cutoff chops off too much
- Too small cutoff leads to collapse
- Plateau appears at intermediate values of the cutoff
- Result does not appear to depend on specific form of the regulator

NEUTRONS

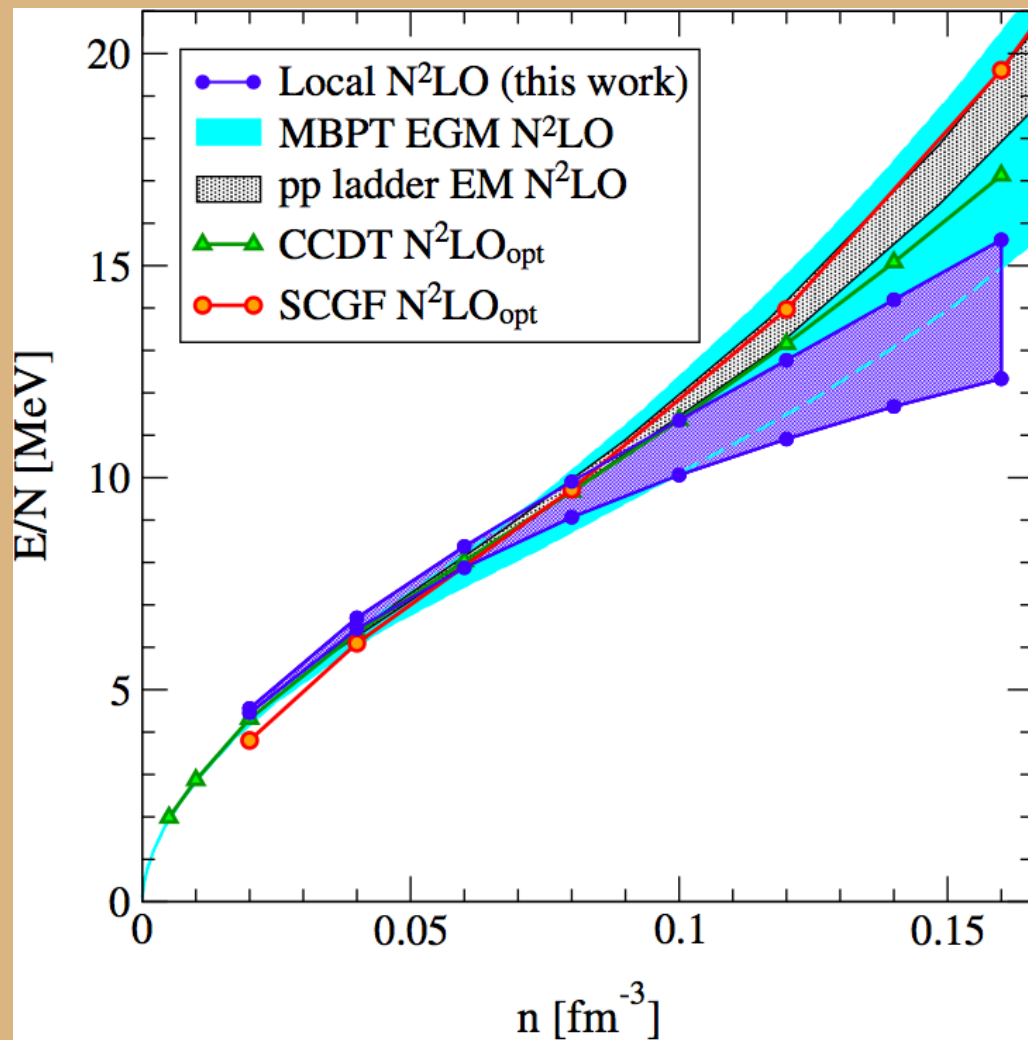
Overall error bands



- NN error band already published
- Now vary 3NF cutoff within plateau
- 3NF cutoff dependence tiny in comparison with NN cutoff one
- 3NF contribution 1-1.5 MeV, cf. with MBPT 4 MeV with EGM

NEUTRONS

Compare with other calculations at N2LO



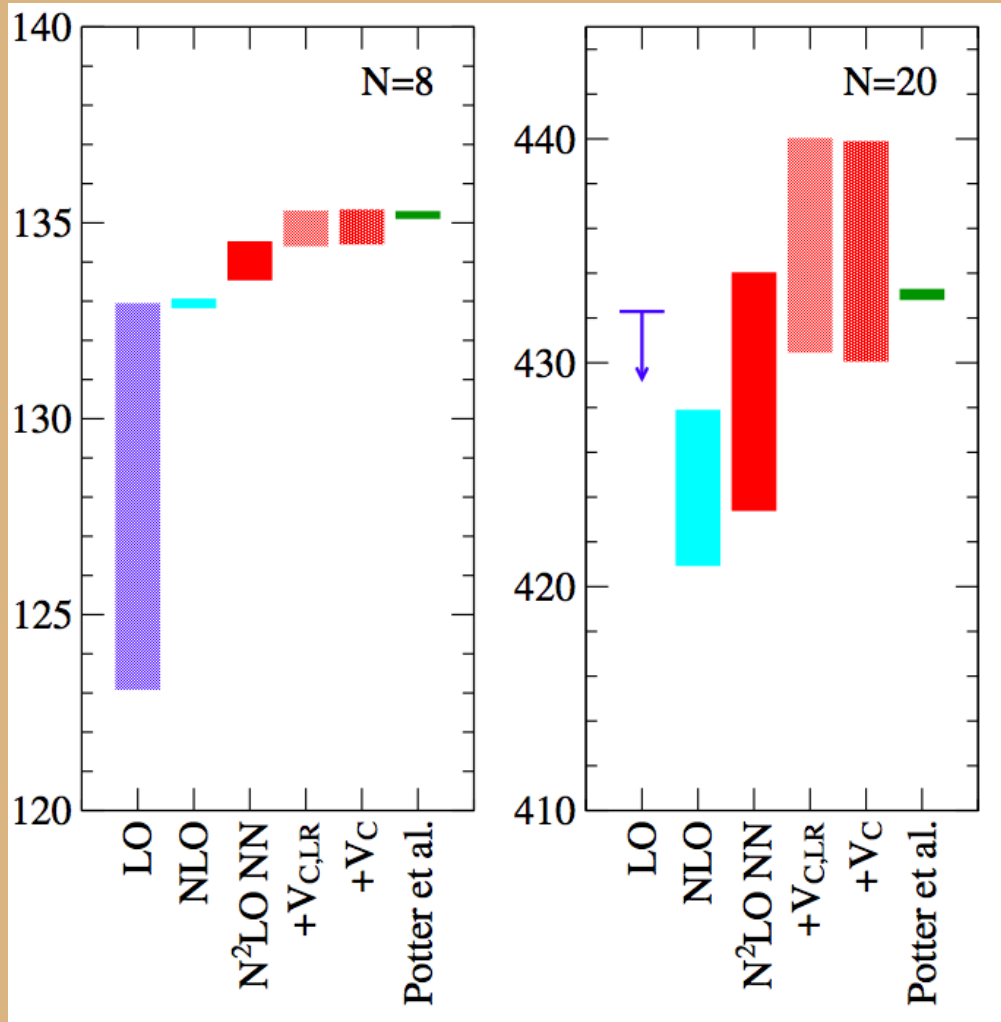
- Overall agreement across methods
- QMC band result of using more than one cutoff
- Band width essentially understood

NEUTRONS

Now turn to neutron drops

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, *in preparation*

Neutron drops with NN+3NF chiral forces



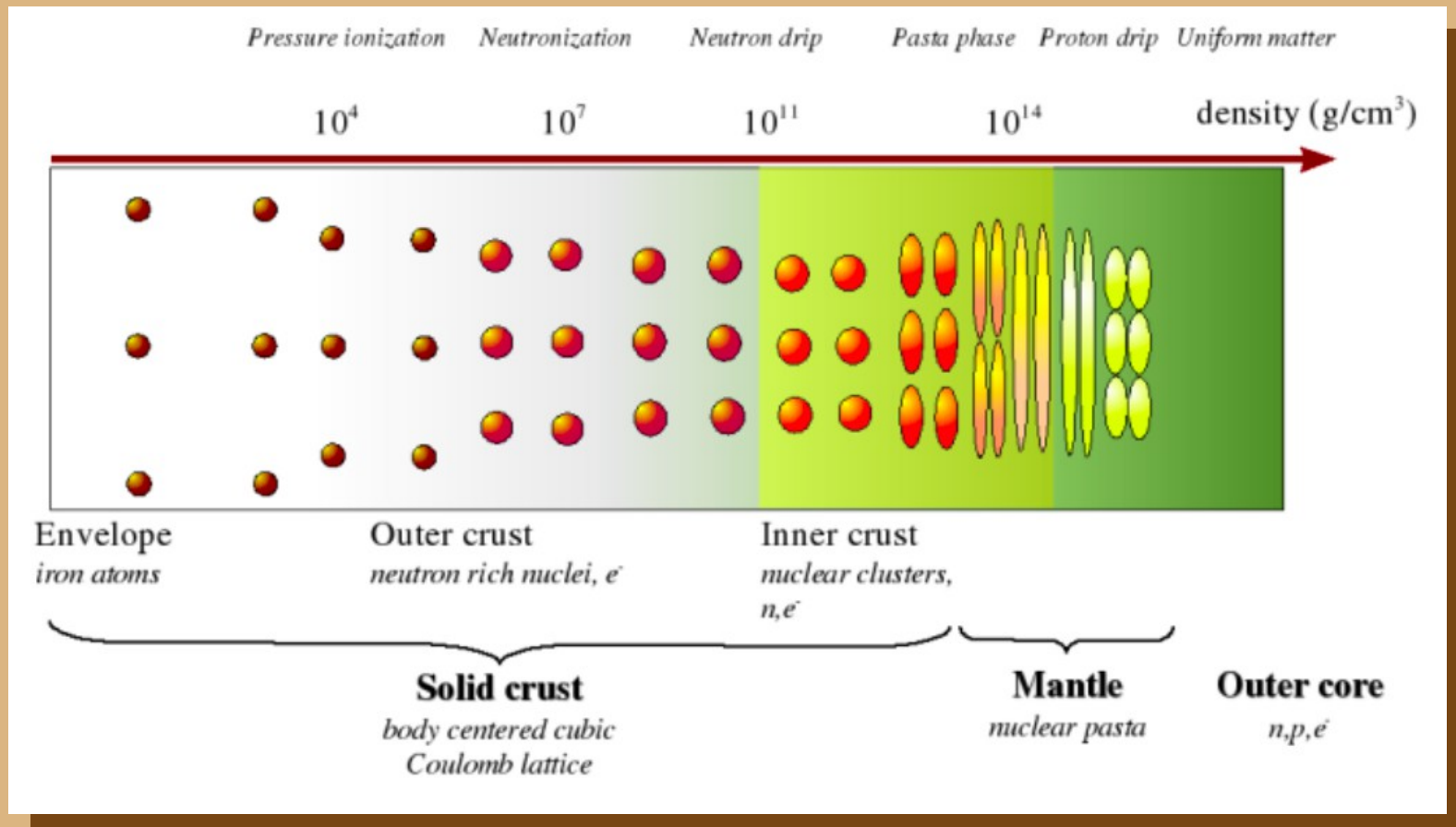
- 10 MeV harmonic oscillator trap
- Order-by-order systematics studied
- Soft LO potential leads to very low energies, especially in larger systems
- Reasonable agreement with Λ CCSD calculations

NEUTRONS

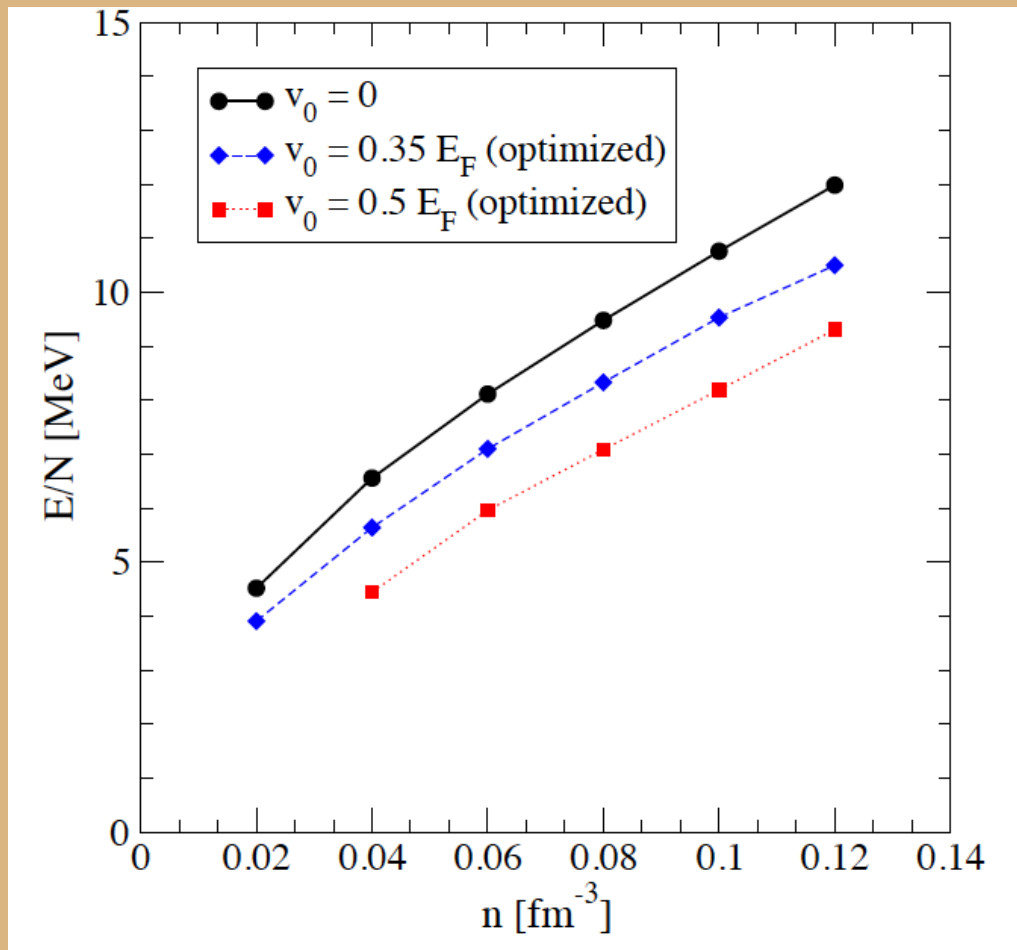
**Remember that neutron-star crusts
also involve a lattice of nuclei**

M. Buraczynski and A. Gezerlis, *in preparation*

Neutron star crusts more than PNM



Static response of neutron matter



- Periodic potential in addition to nuclear forces
- Energy trivially decreased
- Considerable dependence on wave function (physics input)
- Microscopic input for energy-density functionals

NEUTRONS

Conclusions

- Local chiral N²LO 3NF forces derived and being used in the many-body context
- Local 3NF contributions much smaller than non-local ones
- Effects of the regulator intriguing and being further explored
- Static response also being investigated

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- **Ingo Tews** (Darmstadt)